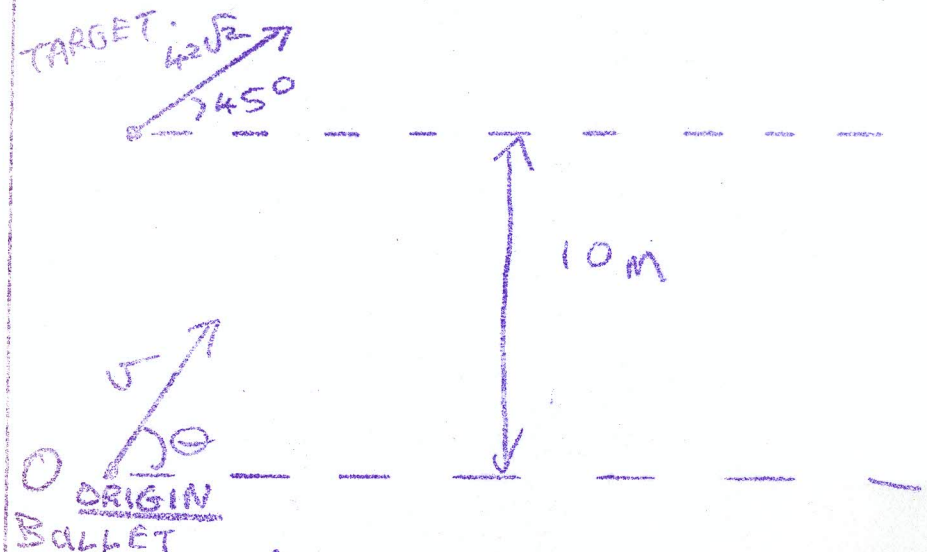
PROJECTILES 1983 (FRONS) *P collision.

Qu.



In general

TARGET.

$\vec{v} = 42\sqrt{2}\cos 45°\,\vec{i} + 42\sqrt{2}\sin 45°\,\vec{j}$

$\vec{v} = 42\vec{i} + 42\vec{j}$

because diagonal bottom

$\vec{r} = 42t\vec{i} + (42t+10)\vec{j}$

BALLET

$\vec{v} = v\cos\theta\,\vec{i} + v\sin\theta\,\vec{j}$

$\vec{r} = v\cos\theta\,t\,\vec{i} + (v\sin\theta\,t - \frac{gt^2}{2})\vec{j}$

When: $v = 70$ and $\tan\theta = \frac{4}{3}$

we have for bullet $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$

$\therefore \vec{v} = 70\frac{3}{5}\vec{i} + (70\frac{4}{5} - 9.8t)\vec{j} =$

$\vec{r}(t) = 70(\frac{3}{5})t\,\vec{i} + (70\frac{4}{5}t - 4.9t^2)\vec{j}$

$\therefore \vec{r}(t) = 42t\,\vec{i} + (56t - 4.9t^2)\vec{j}$.

To collide $\vec{r}(t)$ relative to origin must be same for bullet and target. So.

$42t\,\vec{i} + (42t+10)\vec{j} = 42t\,\vec{i} + (56t - 4.9t^2)\vec{j}$

$\vec{i} \Rightarrow 42t = 42t$ (trivial)

$\vec{j} \Rightarrow 42t + 10 = 56t - 4.9t^2$

$\Rightarrow 4.9t^2 - 14t + 10 = 0$.

$\Rightarrow (7t-10)^2 = 0 \Rightarrow t = \frac{10}{7}$ seconds.

$\therefore$ Horizontal distance $= 42t = 42(\frac{10}{7}) = 60$ m

Min value for θ: Can't assume $v = 70$ as $\tan\theta = \frac{4}{3}$, we are back to the general case.

Obstacle:

$\vec{r}$ FOR TARGET: $42t\,\vec{i} + (42t+10)\vec{j} = v\cos\theta\,t\,\vec{i} + (v\sin\theta\,t - \frac{gt^2}{2})\vec{j}$ $\vec{r}$ FOR BULLET.

$42t = v\cos\theta\,t$ — (1)

and $42t + 10 = v\sin\theta\,t - \frac{gt^2}{2}$

$\Rightarrow \frac{1}{2}gt^2 + 42t + 10 = v\sin\theta\,t$ — (2)

$\frac{(2)}{(1)} \Rightarrow \frac{\frac{1}{2}gt^2 + 42t + 10}{42t} = \frac{v\sin\theta\,t}{v\cos\theta\,t}$

$\Rightarrow 42\tan\theta = \frac{1}{2}gt + 42 + \frac{10}{t}$

To find min of $\tan\theta$ $\frac{d}{dt}$ and put $= 0$.

$\therefore \frac{1}{2}g + 0 - \frac{10}{t^2} = 0 \Rightarrow t^2 = \frac{10}{4.9} \Rightarrow \boxed{t = \frac{10}{7}}$

and this from above $\Rightarrow \boxed{\theta = \tan^{-1}\frac{4}{3}}$ qed.