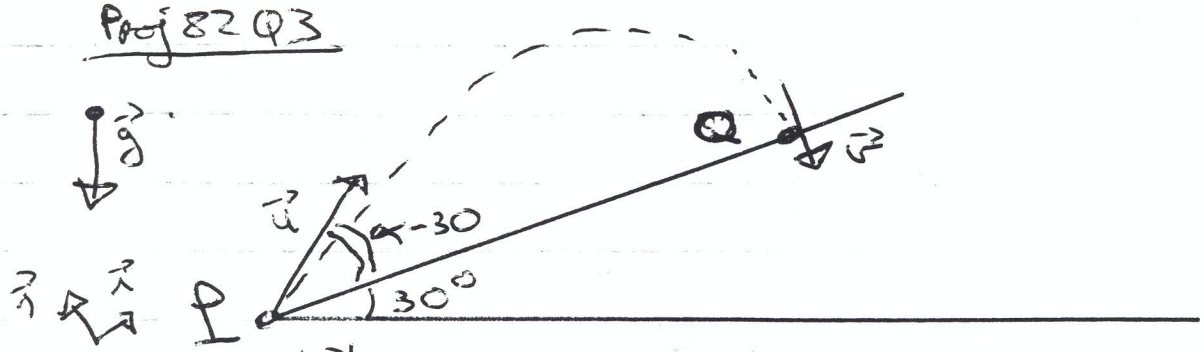
Proj 82 Q3



$|\vec{u}| = 10$.

$\vec{u} = 10\cos(\alpha-30)\vec{i} + 10\sin(\alpha-30)\vec{j}$.

$\vec{g} = -g\sin(30)\vec{i} - g\cos 30\,\vec{j}$. (5)

Velocity at any time: $\vec{v}(t) = [10\cos(\alpha-30) - g\sin 30\,t]\vec{i} + [10\sin(\alpha-30) - g\cos 30\,t]\vec{j}$

Displacement at any time: $\vec{r}(t) = \left[10\cos(\alpha-30)\,t - \frac{g}{2}\sin 30\,t^2\right]\vec{i} + \left[10\sin(\alpha-30)\,t - \frac{g}{2}\cos 30\,t^2\right]$ (5)

Let T be time to travel from P to Q.

At Q geometry $\Rightarrow \vec{v} = \bullet\vec{i} - x\vec{j}$.

$\therefore \vec{i}$ cpts $\Rightarrow 10\cos(\alpha-30) - g\sin 30\,T = 0$.

$\Rightarrow$ $\boxed{T = \dfrac{10\cos(\alpha-30)}{g\sin 30}}$ (1) (10)

As the particle strikes the plane at Q we also have:

At Q $\vec{j}$ cpt of $\vec{r}(t) = 0$. $\Rightarrow$.

$10\sin(\alpha-30)T - \frac{g}{2}\cos 30\,T^2 = 0$

$\Rightarrow$ $10\sin(\alpha-30) - \frac{g}{2}\cos 30\,T = 0$ or $T = 0$

$\Rightarrow$ $\boxed{T = \dfrac{20\sin(\alpha-30)}{g\cos 30}}$ (2) (10)

2nd Expression for $T\tan(\alpha-30)$.

(1) = (2) $\Rightarrow$ $\dfrac{10\cos(\alpha-30)}{g\sin 30} = \dfrac{20\sin(\alpha-30)}{g\cos 30}$

$\dfrac{\sin}{\cos} = \tan \Rightarrow$ $\tan(\alpha-30) \Rightarrow \dfrac{10}{20}\dfrac{\cos 30}{\sin 30} = \dfrac{1}{2}\dfrac{\frac{\sqrt3}{2}}{\frac{1}{2}} \Rightarrow$

$\boxed{\tan(\alpha-30) = \frac{\sqrt3}{2}}$ (5)

2nd Range: Range $= \left(\vec{r}(T)\right)_{\vec{i}}$

$= 10\cos(\alpha-30)T - \frac{g}{2}\sin 30\,T^2$

$\Rightarrow$ Range $= 10\cos(\alpha-30)\dfrac{10\cos(\alpha-30)}{g\sin 30} - \frac{g}{2}\sin 30\left(\dfrac{10\cos(\alpha-30)}{g\sin 30}\right)^2$ (5)

$= \dfrac{100\cos^2(\alpha-30)}{g\sin 30} - \frac{1}{2}\left[\dfrac{100\cos^2(\alpha-30)}{g\sin 30}\right]$

$= \dfrac{100\cos^2(\alpha-30)}{} = \dfrac{100\,\frac{4}{7}}{} = \dfrac{400}{74} = 5\cdot 83$ m (10)

Note: $\cos^2(\alpha-30) = \frac{4}{7}$