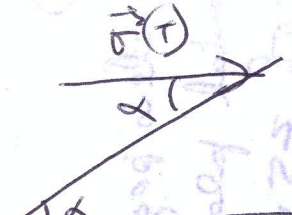
(iii) 1979 Projectiles

Strikes horizontally at Q



$$\Rightarrow \frac{|\vec{v}^2(t)\vec{j}|}{(\vec{v}(t))\vec{v}} = \tan\alpha$$

$$\Rightarrow \left| \frac{u\sin\theta - g\cos\alpha\left[\frac{2u\sin\theta}{g\cos\alpha}\right]}{u\cos\theta - g\sin\alpha\left[\frac{2u\sin\theta}{g\cos\alpha}\right]} \right| = \tan\alpha$$

$$\Rightarrow \frac{|u\sin\theta - 2u\sin\theta|}{u\cos\theta - \tan\alpha\, 2u\sin\theta} = \tan\alpha$$

$$\Rightarrow \frac{|-u\sin\theta|}{u\cos\theta - 2u\sin\theta\tan\alpha} = \tan\alpha$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta - 2\sin\theta\tan\alpha} = \tan\alpha$$

(Change to get $\tan\theta$)

$$\Rightarrow \sin\theta = \cos\theta\tan\alpha - 2\sin\theta\tan^2\alpha.$$

$\div \cos\theta$

$$\Rightarrow \tan\theta = \tan\alpha - 2\tan\theta\tan^2\alpha.$$

$$\Rightarrow \tan\theta + 2\tan\theta\tan^2\alpha = \tan\alpha$$

$$\Rightarrow \tan\theta[1 + 2\tan^2\alpha] = \tan\alpha$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha}{1 + 2\tan^2\alpha}$$

$$\Rightarrow \tan\theta = \frac{\frac{\sin\alpha}{\cos\alpha}}{1 + 2\frac{\sin^2\alpha}{\cos^2\alpha}}$$

$$\Rightarrow \tan\theta = \frac{\sin\alpha\cos^2\alpha}{\cos\alpha\left(1 + 2\frac{\sin^2\alpha}{\cos^2\alpha}\right)\cos^2\alpha}.$$

$$\Rightarrow \tan\theta = \frac{\sin\alpha\cos\alpha}{\cos^2\alpha + 2\sin^2\alpha}.$$

$$\Rightarrow \tan\theta = \frac{\sin\alpha\cos\alpha}{\cos^2\alpha + 2(1-\cos^2\alpha)} \Rightarrow \tan\theta = \frac{\sin\alpha\cos\alpha}{2 - \cos^2\alpha}$$

Very long!