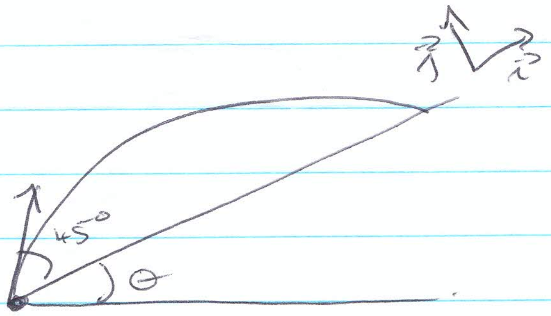
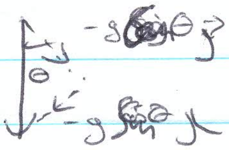
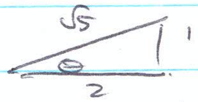
Projectiles Honours

1974





$\theta = \tan^{-1}\frac{1}{2}$.



5

$u = \frac{u}{\sqrt{2}}\vec{i} + \frac{u}{\sqrt{2}}\vec{j}$. $\qquad \cos\theta = \frac{2}{\sqrt{5}} \quad \sin\theta = \frac{1}{\sqrt{5}}$

(i) $\vec{r}(t) = \left(\frac{u}{\sqrt{2}}t - \frac{1}{2}g\sin\theta\, t^2\right)\vec{i} + \left(\frac{u}{\sqrt{2}}t - \frac{1}{2}g\cos\theta\, t^2\right)\vec{j} = \underline{\left(\frac{u}{\sqrt{2}}t - \frac{g}{2\sqrt{5}}t^2\right)\vec{i} + \left(\frac{u}{\sqrt{2}}t - \frac{g}{\sqrt{5}}t^2\right)}$ 5

$\vec{v}(t) = \left(\frac{u}{\sqrt{2}} - g\sin\theta\, t\right)\vec{i} + \left(\frac{u}{\sqrt{2}} - g\cos\theta\, t\right)\vec{j}$.

$\vec{v}(t) = \underline{\left(\frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}}t\right)\vec{i} + \left(\frac{u}{\sqrt{2}} - \frac{2g}{\sqrt{5}}t\right)\vec{j}}$ 5

(ii) ~~Prove~~ Particle strike plane at 90°.

Prove that $(\vec{v})_{\vec{i}}$, for t = time in flight, $\underline{= 0}$. 5

Time in flight: $(\vec{r})_{\vec{j}} = 0$.

$\Rightarrow \frac{u}{\sqrt{2}}t - g\frac{1}{\sqrt{5}}t^2 = 0$

$\Rightarrow t = 0$ OR $\boxed{t = \frac{u\sqrt{5}}{\sqrt{2}\,g}}$ 5

$\therefore (\vec{v})_{\vec{i}}$ for $t = \frac{u\sqrt{5}}{\sqrt{2}\,g} = \frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}}\left[\frac{u\sqrt{5}}{\sqrt{2}\,g}\right]$ 5

$= \frac{u}{\sqrt{2}} - \frac{u}{\sqrt{2}}$

$= \underline{0}$. 5

(iii) Range $= (\vec{r})_{\vec{i}}$ for $t = \frac{u\sqrt{5}}{\sqrt{2}\,g}$

$\Rightarrow$ Range $= \underline{\frac{u}{\sqrt{2}}\left(\frac{u\sqrt{5}}{\sqrt{2}\,g}\right) - \frac{1}{2}g\frac{1}{\sqrt{5}}\left[\frac{u\sqrt{5}}{\sqrt{2}\,g}\right]^2}$ 5

$= \frac{u^2\sqrt{5}}{g\,2} - \frac{g}{2\sqrt{5}}\left[\frac{u^2(5)}{2\,g^2}\right]$

$= \frac{u^2\sqrt{5}}{g\,2} - \frac{u^2\sqrt{5}}{4g}$

$= \frac{u^2\sqrt{5}}{g}\left(\frac{1}{2} - \frac{1}{4}\right)$

$= \frac{u^2\sqrt{5}}{g}\,\frac{1}{4}$