2002 – Relative Velocity Question

2. **(a)** Two boats, B and C, are each moving with constant velocity.
At a certain instant, boat B is 10 km due west of boat C.
The speed and direction of boat B relative to boat C is 2.5 m/s in the direction 60° south of east.

(i) Calculate the shortest distance between the boats, to the nearest metre.

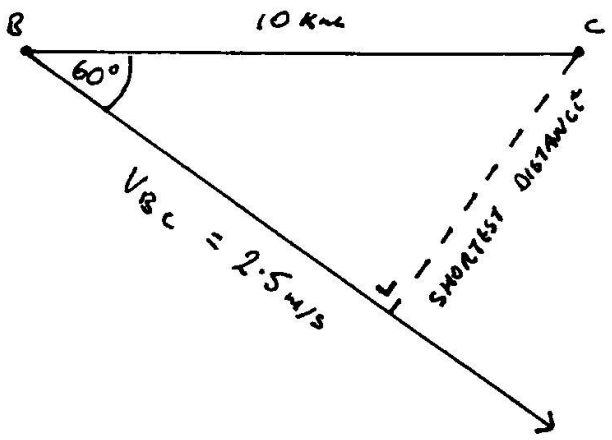
(ii) Calculate the length of time, to the nearest second, for which the boats are less than or equal to 9 km apart.

(b) The velocity of ship P relative to a steady wind is 20 km/hr in the direction 80° north of east.
The velocity of ship Q relative to the same steady wind is 10 km/hr in the direction 20° south of west.

Calculate the magnitude and direction of the velocity of ship P relative to ship Q.
Give your answers to the nearest km and the nearest degree, respectively.

2002

Q2 (a)



(i) SHORTEST DISTANCE:

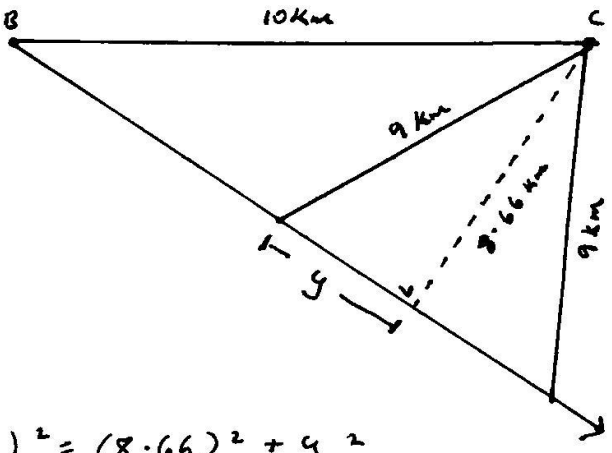
$$\sin 60° = \frac{\text{SHORTEST DIST}}{10{,}000}$$

$$0.866 = \frac{x}{10{,}000}$$

$$(10{,}000)(0.8660) = x$$

$$\underline{\underline{8660\text{ m} = x}}$$

(ii)



FIND y:

$$(9)^2 = (8.66)^2 + y^2$$

$$81 - 75 = y^2$$

$$6 = y^2 \quad \Rightarrow \quad y = \sqrt{6}$$

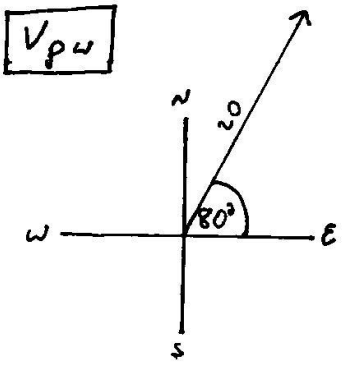
So DISTANCE TRAVELLED WHILE BOATS ARE ≤ 9 km

$$\Rightarrow \quad 2y = 2\sqrt{6}\text{ km} = 4899 \text{ m}$$

$$\text{Time} = \frac{\text{Dist}}{\text{Speed}} = \frac{4899}{2.5} = \underline{\underline{1960 \text{ sec.}}}$$

(b)

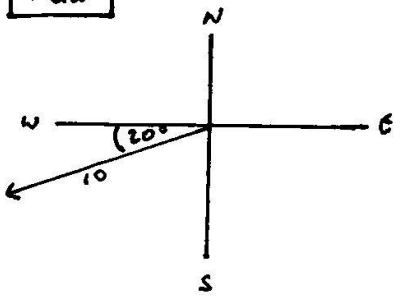
V_{pw}



$$V_{pw} = 20\cos 80°\,\vec{i} + 20\sin 80°\,\vec{j}$$

$$\boxed{V_{pw} = 3.47\,\vec{i} + 19.7\,\vec{j}}$$

V_{qw}



$$V_{qw} = -10\cos 20\,\vec{i} - 10\sin 20\,\vec{j}$$

$$\boxed{V_{qw} = -9.4\,\vec{i} - 3.42\,\vec{j}}$$

$$V_{pw} = V_p + V_w$$

$$\underline{V_{qw} = V_q + V_w \qquad (\times -1)}$$

$$V_{pw} = V_p + V_w$$

$$\underline{-V_{qw} = -V_q - V_w}$$

$$V_{pw} - V_{qw} = V_p - V_q$$

But

$$V_{PQ} = V_P - V_Q$$

so,

$$V_{PW} - V_{QW} = V_{PQ}$$

so

$$V_{PQ} = (3.47\vec{i} + 19.7\vec{j}) - (-9.4\vec{j} - 3.42\vec{j})$$

so,

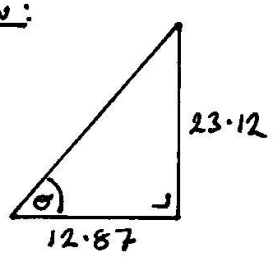
$$V_{PQ} = 12.87\vec{i} + 23.12\vec{j}$$

$$|V_{PQ}| = \sqrt{(12.87)^2 + (23.12)^2}$$

$$|V_{PQ}| = \sqrt{700.17}$$

$$|V_{PQ}| = 26.46 \text{ km} \quad \Rightarrow \quad \underline{|V_{PQ}| = 26 \text{ km/hr}}$$

DIRECTION:



$$\tan\theta = \frac{23.12}{12.87}$$

$$\theta = 61°$$

so, $\underline{\text{Direction} = 61° \text{ N of E}}$