1994 Q10

(a) $\frac{dy}{dx} = \frac{y-xy}{1+x}$, $y=1, x=0$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{1-x}{1+x}\right)$

$\Rightarrow \int \frac{1}{y}\,dy = \int \left(\frac{1-x}{1+x}\right)dx$

$\Rightarrow \ln y = \int \frac{1-x}{1+x}\,dx$

$x+1 \;|\; -x+1 \;|\; -1$

$\frac{-x-1}{2}$ //

$\therefore \int \frac{1-x}{1+x}\,dx = \int \left(-1+\frac{2}{x+1}\right)dx$

$= -x+2\ln(x+1),$

$\therefore \ln y = -x+2\ln(x+1)+C$

// $y=1, x=0$

$\Rightarrow \ln 1 = -0+2\ln(1)+C$

$\Rightarrow 0 = -0+0+C$

$\Rightarrow 0 = C$ //

$\therefore \ln y = -x+2\ln(x+1)$

$\Rightarrow \ln y - 2\ln(x+1) = -x$

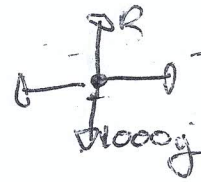
$\Rightarrow \ln y - \ln(x+1)^2 = -x$

$\Rightarrow \ln\left[\frac{y}{(x+1)^2}\right] = -x$

$\Rightarrow \frac{y}{(x+1)^2} = e^{-x}$

$\Rightarrow y = e^{-x}(x+1)^2$

(b) (i) 1500



Power $= T \cdot v$

Power $= 75{,}000$ W $\therefore 75{,}000 = Tv$

$\Rightarrow \frac{75{,}000}{v} = T$

N II $\Sigma F = ma$

$\Rightarrow T - 1500 = 1000a$

$\Rightarrow \frac{75000}{v} - 1500 = 1000a$

$\Rightarrow \frac{75}{v} - 1.5 = a.$

$\Rightarrow \frac{75}{v} - \frac{3}{2} = a$

$\Rightarrow \frac{150-3v}{2v} = a$

(ii) $v=0$, $x=0$, $t=0$; $v=25$, $(x=X)$, $t=T$

Link v and t:

$\frac{dv}{dt} = \frac{150-3v}{2v}$

$\Rightarrow \int_0^{25} \frac{2v}{150-3v}\,dv = \int_0^T dt$

$\int_0^{25} \frac{2v}{150-3v}\,dv = t\Big|_0^T$

$-\frac{3}{2}$

// $-3v+150 \;|\; 2v$

$+2v-100$

$+100$ //

$\Rightarrow \int_0^{25} \frac{2v}{150-3v} = \int_0^{25} -\frac{2}{3} + \frac{100}{150-3v}\,dv$

$= -\frac{2}{3}v + 100\left(-\frac{1}{3}\right)\ln(150-3v)$

$\therefore -\frac{2}{3}v - \frac{100}{3}\ln(150-3v)\Big|_0^{25} = t\Big|_0^T$

$\left(-\frac{2}{3}(25) - \frac{100}{3}\ln(75)\right) - \left(0 - \frac{100}{3}\ln 150\right) = T$

$-\frac{50}{3} + \frac{100}{3}\ln 75 + \frac{100}{3}\ln 150 = T$

$+\frac{50}{3}\left[-1+2\ln\left(\frac{150}{75}\right)\right]$

$\frac{50}{3}\left[-1+2\ln 2\right] = T.$

$6.44 = T$