

1989 Q10 (DE)

1) $x \frac{dy}{dx} = y(x+1)$

$\Rightarrow \frac{dy}{y} = \int \frac{x+1}{x} dx$ (5)

$\Rightarrow \int \frac{dy}{y} = \int (1 + \frac{1}{x}) dx$

$\Rightarrow \ln y = x + \ln x + C$ (3)

= 1, when $x=1 \Rightarrow$

$\ln 1 = 1 + \ln 1 + C$

$0 = 1 + 0 + C$

$-1 = C$ // (5)

$\Rightarrow \ln y = \ln x + x - 1$

$\Rightarrow \ln y - \ln x = x - 1$

$\Rightarrow \ln(\frac{y}{x}) = x - 1$

$\Rightarrow \frac{y}{x} = e^{x-1}$

$\Rightarrow y = x e^{x-1}$ (5)

2) Acceleration \propto square of speed

$\Rightarrow a = -k v^2$ (E) (4)

$x=0 \quad v=6$ $x=35 \quad v=3$



Average speed = $\frac{\text{Distance}}{\text{Time}}$ (3)

Find k (using linking v and x)

(E) $\Rightarrow v \frac{dv}{dx} = -k v^2$ (4)

$\Rightarrow \int \frac{dv}{v} = \int_0^{35} -k dx$ (4)

$\Rightarrow \ln v \Big|_6^3 = -kx \Big|_0^{35}$

$\Rightarrow \ln 3 - \ln 6 = -k(35)$

$\Rightarrow \ln(\frac{1}{2}) = -k(35)$

$\Rightarrow \ln 1 - \ln 2 = -k(35)$

$\Rightarrow 0 - \ln 2 = -k(35)$

$\Rightarrow \frac{\ln 2}{35} = k$ (4)

Find time to go from P to Q

(E) $\Rightarrow \frac{dv}{dt} = -k v^2$ (4)

$\Rightarrow \int_6^3 \frac{dv}{v^2} = \int_0^T -k dt$ (4)

$\Rightarrow -\frac{1}{v} \Big|_6^3 = -kt \Big|_0^T$

$\Rightarrow -\frac{1}{3} - (-\frac{1}{6}) = -kT$

$\Rightarrow -\frac{1}{6} = -kT$

$\Rightarrow \frac{1}{6} = \frac{\ln 2}{35} T$

$\Rightarrow \frac{35}{6 \ln 2} = T$ (4)

$\therefore \text{Average Speed} = \frac{35}{\frac{35}{6 \ln 2}}$

$= 35 \cdot \frac{6 \ln 2}{35}$

$= 6 \ln 2$ (2)

qed.