Kinematics (H) 1985

(Q1) $v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

(i) find t in terms of v and u

$$t = \frac{v-u}{a}$$

Find a in terms of u, v, given $s = 12.5\text{m}$

$\therefore \quad v^2 = u^2 + 2a(12.5)$

$\Rightarrow \quad v^2 - u^2 = 25a$

$\Rightarrow \quad \boxed{a = \frac{v^2-u^2}{25}}$

$\therefore \quad t = \frac{v-u}{\frac{v^2-u^2}{25}} \Rightarrow t = \frac{v-u}{v^2-u^2} \times \frac{25}{1}$

Remember $v^2 - u^2 = (v-u)(v+u) \Rightarrow t = \frac{v-u}{(v-u)(v+u)} \times \frac{25}{1}$

$$\boxed{t = \frac{25}{v+u}}$$

(ii) When time $= \frac{1}{2}t$

new time $= \frac{1}{2}\frac{25}{v+u} = \frac{12.5}{v+u}$

To find the fractional change in length find

$\frac{s}{12.5} = \frac{\text{new length}}{\text{old length}}$

Finds:

$s = ut + \frac{1}{2}at^2$.

$$s = u\frac{12.5}{(v+u)} + \frac{1}{2}\left(\frac{v^2-u^2}{25}\right)\left(\frac{12.5}{v+u}\right)^2$$

$$= \frac{12.5u}{(v+u)} + \frac{(12.5)^2}{50}\frac{v^2-u^2}{(v+u)^2}$$

$$= \frac{12.5u}{(v+u)} + \frac{12.5}{4}\frac{(v-u)(v+u)}{(v+u)(v+u)}$$

$$= \frac{12.5u}{(v+u)} + \frac{12.5}{4}\left(\frac{v-u}{v+u}\right)$$

$$= \frac{12.5}{(v+u)}\left[u + \frac{v}{4} - \frac{u}{4}\right]$$

$$= \frac{12.5}{(v+u)}\left[\frac{3}{4}u + \frac{v}{4}\right]$$

$$= \frac{12.5}{4(v+u)}\left[3u + v\right]$$

So fractional change

$$\frac{s}{12.5} = \frac{12.5}{4(v+u)}\left[3u+v\right] \div 12.5$$

$$\boxed{\text{Change} = \frac{3u+v}{4(v+u)}.}$$