AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

Scéim Mharcála | *Scrúduithe Ardteistiméireachta, 2002*

Matamaitic Fheidhmeach | *Gnáthleibhéal*

Marking Scheme | *Leaving Certificate Examination, 2002*

Applied Mathematics | *Ordinary Level*

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3), 15 (att 5).

2 Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

3 Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4 Number the grid on each script 1 to 9 in numerical order, not the order of answering.

5 Scrutinise **all** pages of the answer book.

6 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. A train stops at stations P and Q which are 2000 metres apart.
The train accelerates uniformly from rest at P, reaching a speed of 20 m/s in 10 seconds.
The train maintains this speed of 20 m/s before decelerating uniformly at 0.5 m/s^2, coming to rest at Q.

(i) Find the acceleration of the train.

(ii) Find the time for which the train is decelerating.

(iii) Find the distance and the time for which the train is travelling at constant speed .

(iv) Draw an accurate speed-time graph of the motion of the train from P to Q.

(i)

$$v = u + at$$
$$20 = 0 + a(10)$$
$$a = 2 \text{ m/s}^2$$

10

(ii)

$$v = u + at$$
$$0 = 20 - 0.5(t)$$
$$t = 40 \text{ s}$$

10

(iii)

accelerating :
$$s = 0(10) + \tfrac{1}{2}(2)(10)^2$$
$$= 100 \text{ m}$$

5

decelerating :
$$s = 20(40) - \tfrac{1}{2}(0.5)(40)^2$$
$$= 400 \text{ m}$$

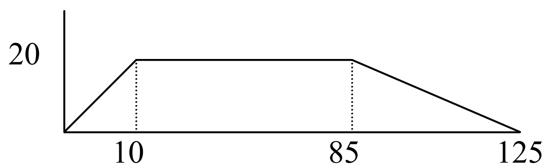
5

constant speed :
$$\text{distance} = 2000 - 100 - 400$$
$$= 1500 \text{ m}$$

$$s = ut + \tfrac{1}{2}at^2$$
$$1500 = 20t + 0$$
$$t = 75 \text{ s}$$

5

(iv)



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2. Ship A is travelling due west with a constant speed of 10 km/hr.
Ship B is travelling at a constant velocity.
At 1200 hours, the radar screen of ship A shows the position of ship B relative to ship A as $-2\vec{i} - 20\vec{j}$ kilometres.
At 1400 hours, two hours later, the position of ship B relative to ship A is $8\vec{i} + 4\vec{j}$ kilometres.

(i) Write down the velocity of ship A in terms of $\vec{i}$ and $\vec{j}$.

(ii) Show that the change in the position of ship B relative to ship A between 1200 hours and 1400 hours is $10\vec{i} + 24\vec{j}$ kilometres.

(iii) Find the velocity of ship B relative to ship A.

(iv) Find the speed and direction of ship B.
Give the direction to the nearest degree.

(i) $V_A = -10\vec{i} + 0\vec{j}$ — 10

(ii) change in position $= (8\vec{i} + 4\vec{j}) - (-2\vec{i} - 20\vec{j})$
$= 10\vec{i} + 24\vec{j}$ — 15

(iii) $V_{BA} = \dfrac{10\vec{i} + 24\vec{j}}{2}$
$= 5\vec{i} + 12\vec{j}$ — 10

(iv) $V_{BA} = V_B - V_A$ — 5
$V_B = V_{BA} + V_A$
$= (5\vec{i} + 12\vec{j}) + (-10\vec{i} + 0\vec{j})$
$= -5\vec{i} + 12\vec{j}$ — 5

speed $= \sqrt{(-5)^2 + (12)^2}$
$= 13$ km/hr

direction $= \tan^{-1}\left(\dfrac{12}{5}\right)$
$= 67°$ north of west — 5

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3. A straight vertical cliff is 80 m high.
Projectile P is fired horizontally directly out to sea from the top of the cliff with a speed of x m/s. Projectile P hits the sea at a distance of 80 m from the foot of the cliff.

(i) Find the time it takes projectile P to hit the sea.

(ii) Find the value of x.

Another projectile, Q, is fired upwards at an angle α to the horizontal and with an initial speed of 15 m/s directly out to sea from the top of the cliff.
Projectile Q takes takes one second longer than projectile P to hit the sea.

(iii) Show that $\sin\alpha = \frac{3}{5}$.

(iv) How far from the foot of the cliff does projectile Q hit the sea?

(i)

$$r_j = -80$$
$$0(t) + \tfrac{1}{2}at^2 = -80$$
$$0 - 5t^2 = -80$$
$$t^2 = 16$$
$$t = 4\text{ s}$$

10, 10

(ii)

$$r_i = 80$$
$$x(t) = 80$$
$$x(4) = 80$$
$$x = 20\text{ m/s}$$

10

(iii)

time = 5 s to hit sea

$$r_j = -80$$
$$u\sin\alpha.(t) - \tfrac{1}{2}gt^2 = -80$$
$$15\sin\alpha.(5) - 5(25) = -80$$
$$75\sin\alpha = 45$$
$$\sin\alpha = \frac{45}{75} = \frac{3}{5}$$

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(iv)

$$\text{Range} = u\cos\alpha.(t)$$
$$= 15\left(\tfrac{4}{5}\right)(5)$$
$$= 60\text{ m}$$

10

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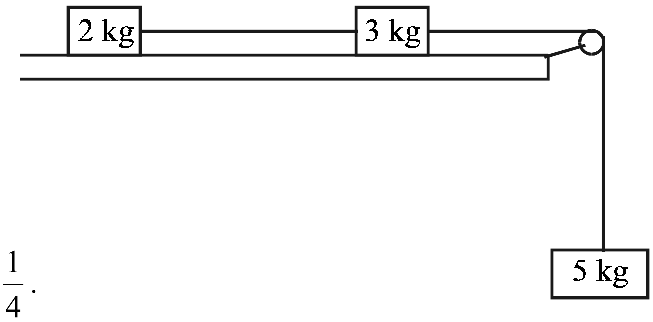
4. Particles, of masses 2 kg and 3 kg, resting on a rough horizontal table, are connected by a light taut inextensible string.
The coefficient of friction between the 2 kg mass and the table is $\frac{1}{8}$ and between the 3 kg mass and the table is $\frac{1}{4}$.



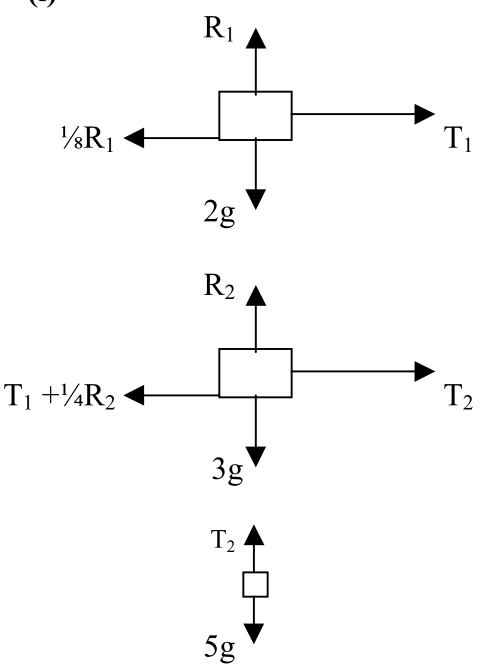
The 3 kg mass is connected by a second light inextensible string passing over a smooth light pulley at the edge of the table to a particle of mass 5 kg.
The 5 kg mass hangs freely under gravity.
The particles are released from rest.
The 5 kg mass moves vertically downwards.

(i) Show on separate diagrams all the forces acting on each particle.

(ii) Write down the equation of motion for each particle.

(iii) Find the common acceleration of the particles and the tension in each string.

(i)



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(ii)

$$T_1 - \tfrac{1}{8}(2g) = 2a \qquad 5$$
$$T_2 - T_1 - \tfrac{1}{4}(3g) = 3a \qquad 5$$
$$5g - T_2 = 5a \qquad 5$$

(iii)

$$5g - \tfrac{1}{4}g - \tfrac{3}{4}g = 10a$$
$$a = \tfrac{4}{10}g \quad \text{or} \quad 4\ \text{m/s}^2 \qquad 5$$

$$\Rightarrow T_1 = 10.5 \text{ and } T_2 = 30 \qquad 5$$

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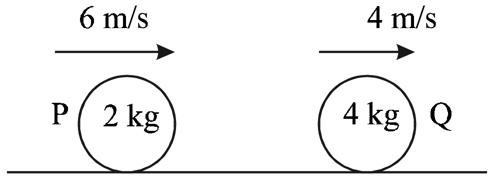
5. A smooth sphere P, of mass 2 kg, moving with a speed of 6 m/s collides directly with a smooth sphere Q, of mass 4 kg, moving in the same direction with a speed of 4 m/s on a smooth horizontal table.



The coefficient of restitution for the collision is $\frac{1}{2}$.

(i) Find the speed of P and the speed of Q after the collision.

(ii) Find the loss in kinetic energy due to the collision.

(i)

PCM $\quad 2(6)+4(4) = 2v_1 + 4v_2$ — 10

NEL $\quad v_1 - v_2 = -\frac{1}{2}(6-4)$ — 10

$$2v_1 + 4v_2 = 28$$
$$v_1 - v_2 = -1$$

$$v_1 + 2v_2 = 14$$
$$2v_1 - 2v_2 = -2$$

$v_1 = 4$ and $v_2 = 5$ — 10

30

(ii)

$$\text{KE before} = \tfrac{1}{2}(2)(6^2) + \tfrac{1}{2}(4)(4^2)$$
$$= 36 + 32$$
$$= 68$$ — 10

$$\text{KE after} = \tfrac{1}{2}(2)(4^2) + \tfrac{1}{2}(4)(5^2)$$
$$= 66$$ — 5

$$\text{Loss in KE} = 68 - 66$$
$$= 2\text{ J}$$ — 5

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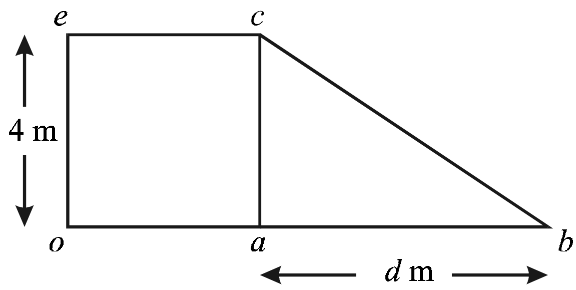
6. **(a)** Particles of weight 3 N, 2 N, 1 N and 4 N are placed at the points (−2, −3), (2, −1), (1, 5) and (x, y), respectively.
The centre of gravity of the four particles is at the origin.

Find the value of x and the value of y.

(b) A uniform lamina *obce* consists of a square *oace* with side of length 4 m and a right-angled triangle *abc* with $|ab| = d$ m.



Taking *o* as the origin and *oe* as the direction of the y axis, the y co-ordinate of the centre of gravity of the lamina is $\frac{12}{7}$.

(i) Calculate the value of d.

(ii) Find the x co-ordinate of the centre of gravity of the lamina, giving your answer as a fraction.

(a)

$$10(0) = 4x + 1(1) + 2(2) + 3(-2)$$
$$x = \tfrac{1}{4}$$ 10

$$10(0) = 4y + 1(5) + 2(-1) + 3(-3)$$
$$y = \tfrac{3}{2}$$ 10

(b)

$$\text{square: area} = 16$$
$$\text{triangle: area} = \tfrac{1}{2}(\text{d})(4) = 2\text{d}$$
$$\text{lamina: area} = 16 + 2\text{d}$$ 10

$$\text{square: c.g.} = (2, 2)$$
$$\text{triangle: c.g.} = \{4 + \tfrac{1}{3}\text{d}, \tfrac{4}{3}\}$$
$$\text{lamina: c.g.} = \{\text{x}, \tfrac{12}{7}\}$$ 10

$$\{16\}(2) + 2d(\tfrac{4}{3}) = (16 + 2d)(\tfrac{12}{7})$$
$$d = 6\,\text{cm}$$ 5

$$\{16\}(2) + 12(4 + 2) = (16 + 12)(x)$$
$$x = \frac{104}{28} = \frac{26}{7} \quad \text{cm}$$ 5

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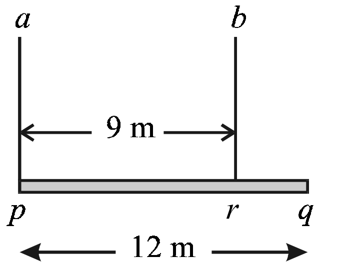
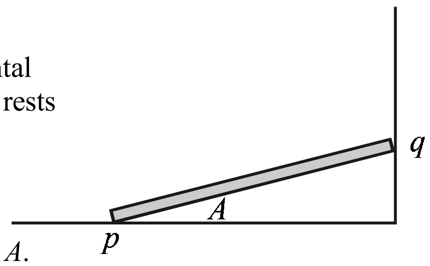
7. A uniform beam, $[pq]$, of mass 12 kg and length 12 m, is held in a horizontal position by two vertical light inelastic strings.
One string is attached from a fixed point a to the end p of the beam.
The other string is attached from a fixed point b to a point r on the beam, where $|pr| = 9$ m.



(i) Find the value of the tension in the string $[rb]$.

(ii) Find the value of the tension in the string $[ap]$.

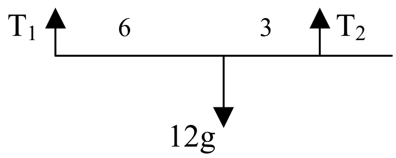
The two strings are removed from the beam, $[pq]$.
The beam is now placed with its end p on rough horizontal ground, where the coefficient of friction is 1. The end q rests against a rough vertical wall where the coefficient of friction is $\frac{1}{2}$.



The angle of inclination of the beam to the horizontal is A.
The normal reaction at p is 80 N and the normal reaction at q is 80 N.

(iii) Show that $\tan A = \frac{1}{4}$.

(i)



$$T_2\{9\} = 12g\{6\}$$

$$\Rightarrow \quad T_2 = 8g \text{ or } 80\text{ N}$$ 15

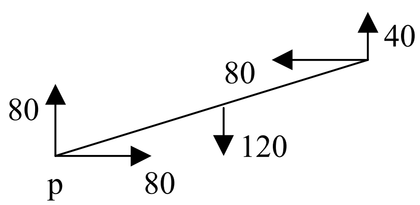
(ii)

$$T_1 + T_2 = 120$$ 5

$$T_1 + 80 = 120$$ 5

$$\Rightarrow \quad T_1 = 40\text{ N}$$ 5

(iii)



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Moments about p :

$$80\,(12\sin A) + 40\,(12\cos A) = 120(6\cos A)$$ 10

$$24\sin A = 6\cos A$$

$$\Rightarrow \quad \tan A = \tfrac{1}{4}$$ 5

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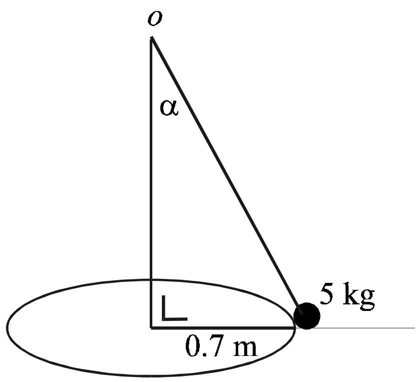
8. A particle of mass 5 kg describes a horizontal circle of radius 0.7 metres with constant angular velocity ω radians per second on a smooth horizontal table.
The particle is connected by means of a light inextensible string to a fixed point o which is vertically above the centre of the circle.
The inclination of the string to the vertical is α, where $\tan\alpha = \frac{1}{2}$.



The tension in the string is T newtons,
the normal reaction between the particle and the table is R newtons
and $R = T\sqrt{5}$.

(i) Write down the value of $\sin\alpha$ and the value of $\cos\alpha$.

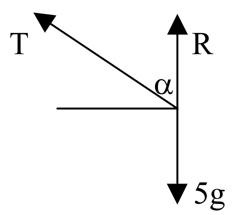
(ii) Show on a diagram all the forces acting on the particle.

(iii) Find the value of T and the value of R.

(iv) Find the value of ω.

(i) $\sin\alpha = \frac{1}{\sqrt{5}}$ and $\cos\alpha = \frac{2}{\sqrt{5}}$ 10

(ii)



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(iii) $T\cos\alpha + R = 5g$ 5

$$T\left\{\frac{2}{\sqrt{5}}\right\} + T\sqrt{5} = 50$$ 5

$$\Rightarrow T = \frac{50\sqrt{5}}{7}$$ 5

$$\Rightarrow R = \frac{250}{7}$$ 5

(iv) $T\sin\alpha = mr\omega^2$ 5

$$\left\{\frac{50\sqrt{5}}{7}\right\}\left(\frac{1}{\sqrt{5}}\right) = 5(0.7)\omega^2$$

$$\omega = \frac{10}{7}$$ 5

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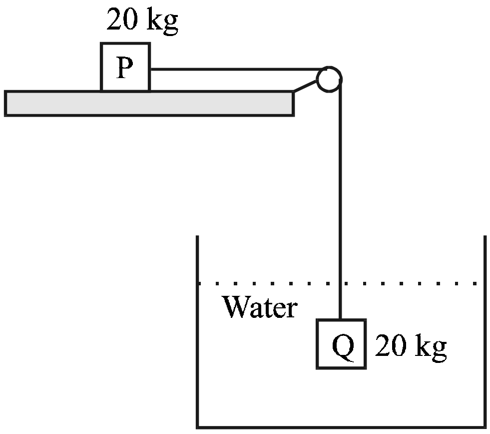
9. Two identical blocks, P and Q, are connected by means of a light inextensible string passing over a smooth light pulley at the edge of a rough horizontal table.
Each block is a cube with side of length 0.2 m and mass 20 kg.
The coefficient of friction between block P and the table is μ.

Block P is at rest on the table.
Block Q is immersed in water in a tank.
The system is in equilibrium and block P is on the point of slipping.



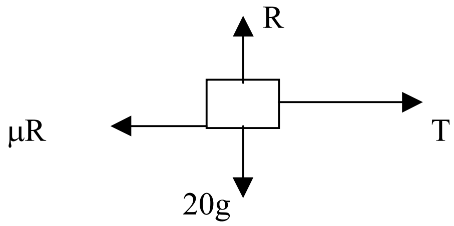
(i) Show, on separate diagrams, all the forces acting on each block.

(ii) Find the value of the tension in the string.

(iii) Find the value of μ.

[Density of water = 1000 kg/m^3.]

(i)



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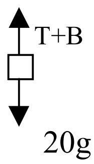

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(ii)

$B = \rho V g$ 5

$= 1000(0.2^3)(10)$ 5

$= 80$

$T + B = 20g$ 5

$T = 200 - 80$

$= 120\text{ N}$ 5

(iii)

$\mu R = T$ 5

$\mu(200) = 120$

$\mu = \frac{120}{200} = \frac{3}{5}$ or 0.6 5

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