



Coimisiún na Scrúduithe Stáit State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2005

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not clearly shown.

1. (a) Car A and car B travel in the same direction along a horizontal straight road. Each car is travelling at a uniform speed of 20 m/s. Car A is at a distance of d metres in front of car B. At a certain instant car A starts to brake with a constant retardation of 6 m/s^2 . 0.5 s later car B starts to brake with a constant retardation of 3 m/s^2 .

Find

- (i) the distance travelled by car A before it comes to rest
(ii) the minimum value of d for car B not to collide with car A.

- (b) A mass of 8 kg falls freely from rest. After 5 s the mass penetrates sand. The sand offers a constant resistance and brings the mass to rest in 0.01 s.

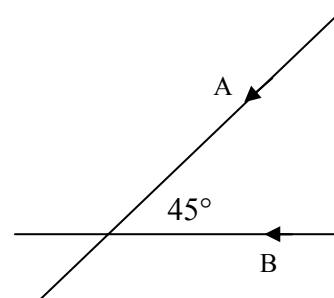
Find

- (i) the constant resistance of the sand
(ii) the distance the mass penetrates into the sand.

2. (a) A woman can swim at u m/s in still water. She swims across a river of width d metres. The river flows with a constant speed of v m/s parallel to the straight banks, where $v < u$. Crossing the river in the shortest time takes the woman 10 seconds.

Find, in terms of u and v , the time it takes the woman to cross the river by the shortest path.

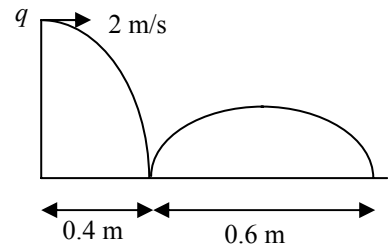
- (b) Two straight roads intersect at an angle of 45° . Car A is moving towards the intersection with a uniform speed of p m/s. Car B is moving towards the intersection with a uniform speed of 8 m/s. The velocity of car A relative to car B is $-2\vec{i} - 10\vec{j}$, where \vec{i} and \vec{j} are unit perpendicular vectors in the east and north directions, respectively.



At a certain instant car A is $220\sqrt{2}$ m from the intersection and car B is 136 m from the intersection.

- (i) Find the value of p .
(ii) How far is car A from the intersection at the instant when the cars are nearest to each other?
Give your answer correct to the nearest metre.

3. (a) A ball is projected horizontally from a point q above a smooth horizontal plane with speed 2 m/s. The ball first hits the plane at a point whose horizontal displacement from q is 0.4 m. The ball next strikes the plane at a horizontal displacement of 1 m from q . The coefficient of restitution between the ball and the plane is e .



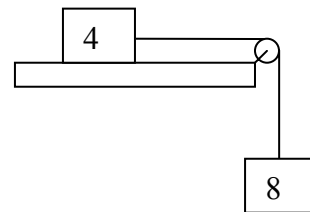
Find the value of e .

- (b) A plane is inclined at an angle β to the horizontal. A particle is projected up the plane with initial velocity u at an angle α to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.

- (i) Find the range of the particle on the inclined plane in terms of u , α and β .
- (ii) Show that for a constant value of u the range is a maximum when

$$\alpha = 45^\circ - \frac{\beta}{2}.$$

4. (a) A particle of mass 4 kg rests on a rough horizontal table. It is connected by a light inextensible string which passes over a smooth, light, fixed pulley at the edge of the table to a particle of mass 8 kg which hangs freely under gravity.



The coefficient of friction between the 4 kg mass and the table is $\frac{1}{4}$.

The system starts from rest and the 8 kg mass moves vertically downwards.

Find

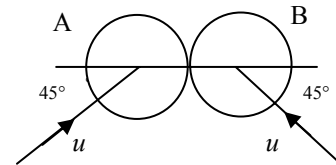
- (i) the tension in the string
- (ii) the force exerted by the string on the pulley.
- (b) Two particles of masses 3 kg and 5 kg are connected by a light inextensible string, of length 4 m, passing over a light smooth peg of **negligible radius**. The 5 kg mass rests on a smooth horizontal table. The peg is 2.5 m directly above the 5 kg mass. The 3 kg mass is held next to the peg and is allowed to fall vertically a distance 1.5 m before the string becomes taut.
- (i) Show that when the string becomes taut the speed of each particle is $\frac{3\sqrt{3g}}{8}$ m/s.
- (ii) Show that the 3 kg mass will not reach the table.

5. (a) Three identical smooth spheres P, Q and R, lie at rest on a smooth horizontal table with their centres in a straight line. Q is between P and R. Sphere P is projected towards Q with speed 2 m/s. Sphere P collides directly with Q and then Q collides directly with R.

The coefficient of restitution for all of the collisions is $\frac{3}{4}$.

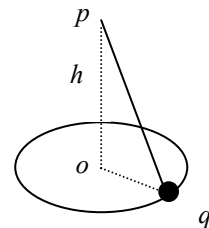
Show that P strikes Q a second time.

- (b) A smooth sphere A, of mass m , moving with speed u , collides with an identical smooth sphere B moving with speed u . The direction of motion of A, before impact, makes an angle 45° with the line of centres at impact. The direction of motion of B, before impact, makes an angle 45° with the line of centres at impact. The coefficient of restitution between the spheres is e .



- (i) Find, in terms of e and u , the speed of each sphere after the collision.
- (ii) If $e = \frac{1}{2}$, show that after the collision the angle between the directions of motion of the two spheres is $\tan^{-1}\left(\frac{4}{3}\right)$.

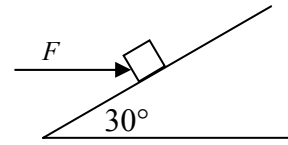
6. (a) A conical pendulum consists of a light inelastic string $[pq]$, fixed at the end p , with a particle attached to the other end q . The particle moves uniformly in a horizontal circle whose centre o is vertically below p . If $|po| = h$, find the period of the motion in terms of h .



- (b) A light elastic string of natural length a and elastic constant k is fixed at one end to a point o on a smooth horizontal table. A particle of mass m is attached to the other end of the string. Initially the particle is held at rest on the table at a distance $2a$ from o , and is then released.

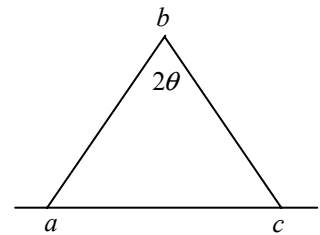
Show that the time taken for the particle to reach o is $\sqrt{\frac{m}{k}}\left(1 + \frac{\pi}{2}\right)$.

7. (a) A particle of weight 100 N lies on a plane. The plane is inclined at 30° to the horizontal. A horizontal force F is applied to the particle. The coefficient of friction between the particle and the inclined plane is $\frac{3}{5}$.



Find the least value of F that will move the particle up the plane.

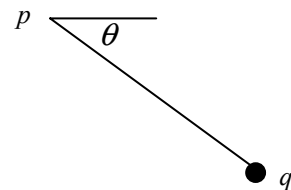
- (b) Two uniform rods, $[ab]$ and $[bc]$, of equal length, are smoothly jointed at b . They rest in a vertical plane with a and c on rough horizontal ground and $|\angle abc| = 2\theta$. The weight of the rod $[ab]$ is $2W$ and the weight of the rod $[bc]$ is W . The coefficient of friction at both a and c is μ .



Find the least value of μ , in terms of θ , necessary for equilibrium.

8. (a) Prove that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through its centre perpendicular to the rod is $\frac{1}{3}ml^2$.

- (b) A uniform rod $[pq]$, of mass $9m$ and length $2l$, has a particle of mass $2m$ attached at q . The system is free to rotate about a smooth horizontal axis through p . The rod is held in a horizontal position and is then given an initial angular velocity $\sqrt{\frac{3g}{2l}}$ downwards.



The diagram shows the rod $[pq]$ when it makes an angle θ with the horizontal.

- (i) Show that when the rod makes an angle θ below its initial horizontal position, its angular velocity is

$$\sqrt{\frac{g(15 + 13 \sin \theta)}{10l}}$$

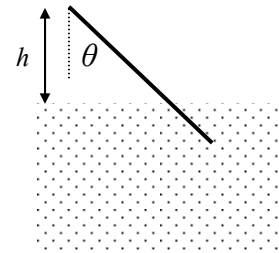
- (ii) Hence, or otherwise, show that the rod performs complete revolutions about p .

9. (a) An alloy is made of iron and aluminium.
A piece of the alloy has a mass of 0.441 kg and a volume of 75 cm^3 .
The relative density of iron is 8 and the relative density of aluminium is 2.7.

Find

- (i) the volume of iron in the piece of alloy
(ii) the mass of aluminium in the piece of alloy.

- (b) A uniform rod of length ℓ and relative density s can turn smoothly about its upper end which is fixed at a height h above the surface of water. The rod is inclined at an angle θ to the vertical and is partially immersed in the water. The rod is at rest.



Show that $\cos \theta = \frac{h}{\ell \sqrt{1-s}}$, where $s < 1$.

10. (a) Solve the differential equation

$$x \frac{dy}{dx} - xy - y = 0$$

given that $y = 1$ when $x = 1$.

- (b) A mass of 9 kg is suspended at the lower end of a light vertical rope. Initially the mass is at rest. The mass is pulled up vertically with an initial pull on the rope of 137.2 N. The pull diminishes uniformly at the rate of 1 N for each metre through which the mass is raised.

- (i) Show that the resultant upward force on the mass when it is x metres above its initial position is

$$49 - x.$$

- (ii) Find the speed of the mass when it has been raised 15 metres.
(iii) Find the work done by the pull on the rope when the mass has been raised by 15 m.