

APPLIED MATHEMATICS - HIGHER LEVEL

WEDNESDAY 23 JUNE - AFTERNOON 2.00 TO 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

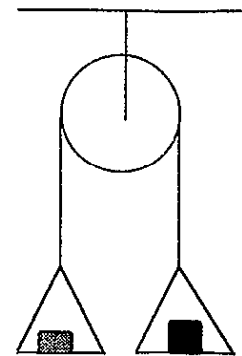
**Marks may be lost if necessary work is not clearly shown
or you do not indicate where a calculator has been used.**

1. (a) A car of mass 1500 kg travels up a slope of gradient $\sin^{-1}\left(\frac{1}{50}\right)$ against a constant resistance of 0.2 N per kilogram. Find
- (i) the constant force required to produce an acceleration of 0.1 m/s^2 .
 - (ii) the power which is developed when the speed is 20 m/s.
- (b) A particle travels in a straight line with constant acceleration f for $2t$ seconds and covers 15 metres. The particle then travels a further 55 metres at constant speed in $5t$ seconds. Finally the particle is brought to rest by a constant retardation $3f$.
- (i) Draw a speed-time graph for the motion of the particle.
 - (ii) Find the initial velocity of the particle in terms of t .
 - (iii) Find the total distance travelled in metres, correct to two decimal places.
2. (a) An aeroplane has a speed of 160 m/s in still air. When the wind blows from the east, the velocity of the aeroplane as observed from the ground is 120 m/s towards the north-east. Find the speed of the wind correct to two decimal places.
- (b) Two ships A and B move with constant speeds 48 km/h and 60 km/h respectively. At a certain instant ship B is 30 km west of A and is travelling due south. Find
- (i) the direction ship A should steer in order to get as close as possible to ship B
 - (ii) the shortest distance between the ships.

3. A particle is projected from a point p up an inclined plane with a speed of $4g\sqrt{2}$ m/s at an angle $\tan^{-1}\left(\frac{1}{3}\right)$ to the inclined plane. The plane is inclined at an angle θ to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle is moving horizontally when it strikes the plane at the point q .

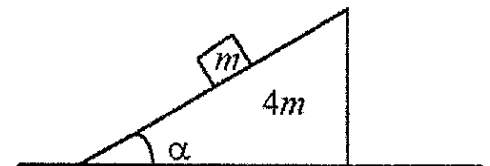
- (a) Find the two possible values for θ .
- (b) If $\tan \theta = 0.5$ then
- (i) find the magnitude of the velocity with which the particle strikes the inclined plane at q .
- (ii) determine the total energy at p and show that it is equal to the total energy at q .

4. (a) Two scale-pans each of mass 0.5 kg are connected by a light inelastic string which passes over a smooth light fixed pulley. A mass of 0.2 kg is placed on one pan and a mass of 0.4 kg is placed on the other pan. The system is released from rest. Calculate



- (i) the acceleration of the system
- (ii) the forces between the masses and the pans.

- (b) A smooth wedge of mass $4m$ and slope α , is placed on a smooth horizontal surface. A particle of mass m moves down the inclined face of the wedge.

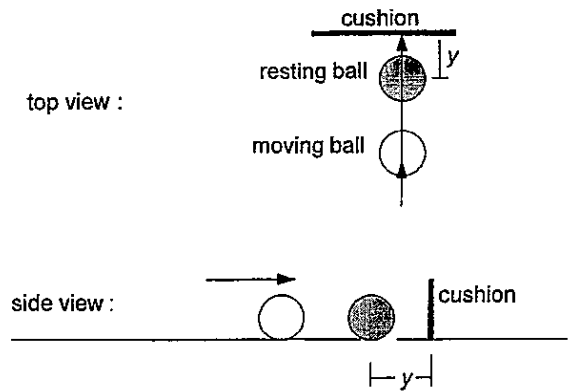


- (i) Show on separate diagrams, the forces acting on the wedge and on the particle.
- (ii) Prove that the acceleration of the wedge is

$$\frac{g \cos \alpha \sin \alpha}{4 + \sin^2 \alpha}$$

- (iii) If $\alpha = 30^\circ$ find the acceleration of the mass relative to the wedge.

5. (a) A smooth sphere moves on a horizontal table. It strikes an identical sphere at rest on the table. The latter is at a distance y from a vertical cushion. The impact is along the line of centres and normal to the cushion. The next collision between the spheres takes place at a distance d from the cushion.



- (i) Prove that $d = \frac{2e^2 y}{1 + e^2}$, where e is the coefficient of restitution for impacts between spheres and between a sphere and the cushion.

- (ii) Interpret the result when $e = 1$.

- (b) Two equal smooth spheres A and B collide. The velocity of A before the collision is $3\sqrt{3}\mathbf{i} + 3\mathbf{j}$ and the velocity of B before the collision is $\frac{1}{2}(-u\sqrt{3}\mathbf{i} + u\mathbf{j})$ where \mathbf{i} and \mathbf{j} are unit perpendicular vectors along and perpendicular to the line of centres, respectively. The velocity of A after the collision is $\frac{1}{2}(-v\mathbf{i} + v\sqrt{3}\mathbf{j})$.

If the coefficient of restitution is 0.7, find the magnitude and direction of the velocity of sphere B after the collision.

6. (a) A particle moves with simple harmonic motion of period $\frac{\pi}{2}$. Initially it is 8 cm from the centre of motion and moving away from the centre with a speed of $4\sqrt{2}$ cm/s. Find an equation for the position of the particle in time t seconds.

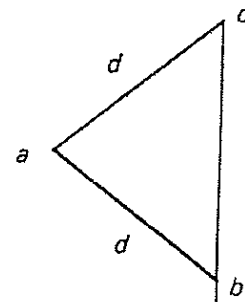
- (b) A particle of mass 0.5 kg at rest on a smooth horizontal table is attached to two points p and q , which are 1.2 m apart, by two light elastic strings. The string attached to p has a natural length 0.4 m and elastic constant 75 N/m. The string attached to q has a natural length 0.6 m and elastic constant 50 N/m.



- (i) Find the equilibrium position.
- (ii) Prove that if the particle is displaced in the direction $p\vec{q}$, through such a distance that neither string goes slack and is then released, it moves with simple harmonic motion.

7. (a) A particle of weight W rests on a rough plane inclined at 26° to the horizontal. P is the least force, acting up along the plane needed to move the particle up the plane. Prove that if P is less than W then the angle of friction is less than 32° .

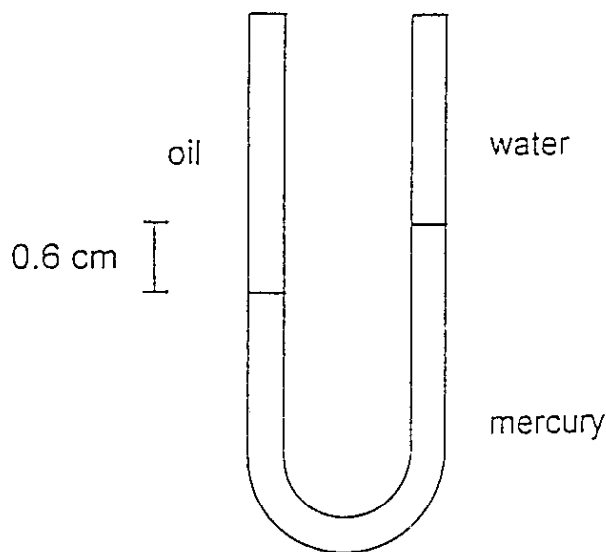
- (b) A uniform rod $[ab]$ of length d rests with end b against a rough vertical wall. The other end a is tied to a point c by a light string $[ac]$ of length d . If the coefficient of friction between the rod and the wall is $\frac{1}{2}$, find the least angle that the rod can make with the wall.



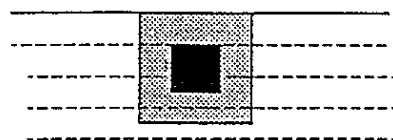
8. (a) Prove that the moment of inertia of a uniform rod [ab] of mass m and length $2l$ about an axis through its centre perpendicular to the rod is $\frac{1}{3} ml^2$.
- (b) A uniform rod of mass m is free to rotate in a vertical plane about an axis which is perpendicular to the rod and 0.32 m from its centre of gravity. For small oscillations the rod has the same period as a simple pendulum of length 0.5 m.
- (i) Find the length of the rod.
- (ii) For what other distance between the axis and the centre of gravity will the period be the same?
- (iii) Where must the axis be located to give a minimum period?

9. (a) A U-tube whose limbs are vertical and of equal length has mercury poured in until the level is 50 cm from the top in each limb. Water is poured into one limb and oil into the other until the U-tube is filled. The difference in height of the mercury levels is 0.6 cm. [Diagram not to scale].

If the relative density of mercury is 13.6, find the relative density of the oil correct to two places of decimals.



- (b) A cubical block of ice of edge 5 cm and relative density 0.9 just floats in water with a piece of iron of relative density 8 embedded in it.



Find the mass of the iron.

10. (a) Solve the differential equation

$$\left(\frac{7}{v^2 + 1} \right) \frac{dv}{dx} = \frac{1}{x}$$

given that $v = 0$ when $x = 1$.

- (b) The rocket engine of a 12 tonne missile produces a thrust of 180.1 kN. The missile is launched in a vertical direction. The air resistance is v^2 N where v is the speed of the missile.
- (i) Find the speed of the missile after 30 seconds.
- (ii) Find the percentage error in this speed if air resistance is ignored.