AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1996

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 21 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

1. *(a)* A particle starts from rest and moves in a straight line with uniform acceleration. It passes three points a, b and c where $|ab| = 105$ m and $|bc| = 63$ m. If it takes 6 seconds to travel from a to b and 2 seconds to travel from b to c find

(i) its acceleration

(ii) the distance of a from the starting position.

(b) A lift starts from rest with constant acceleration 4 m/s^2. It then travels with uniform speed and finally comes to rest with constant retardation 4 m/s^2. The total distance travelled is d and the total time taken is t.

(i) Draw a speed-time graph.

(ii) Show that the time for which it travelled with uniform speed is

$$\sqrt{t^2 - d}.$$

2. A ship, B, is travelling due West at 25.6 km/h. A second ship, C, travelling at 32 km/h is first sighted 17 km due North of B. From B the ship C appears to be moving South-east.

Find

(i) the direction in which C is actually moving

(ii) the velocity of C relative to B

(iii) the shortest distance between the ships in the subsequent motion

(iv) the time that elapses, after first sighting, before the ships are again 17 km apart.

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3. *(a)* A particle is projected from the ground with a velocity of 50.96 m/s at an angle $\tan^{-1}\frac{5}{12}$ to the horizontal. On its upward path it just passes over a wall 14.7 m high. During its flight it also passes over a second wall 18.375 m high.

Show that the second wall must be not less than 23.52 m and not more than 70.56 m from the first wall.

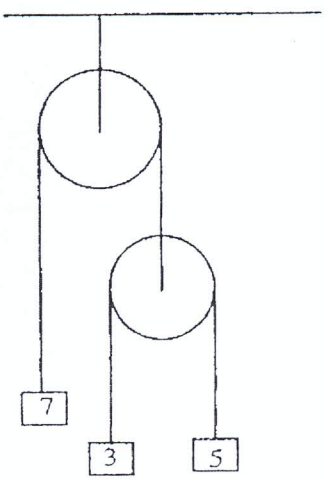
(b) A plane is inclined at an angle of 2β to the vertical. A particle is projected up the plane with initial velocity $u\cos\beta$ at an angle β to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.

Show

(i) that the time of flight of the particle is $\frac{u}{g}$

(ii) that the range of the particle on the plane is $\frac{u^2}{2g}$.

4. A light inextensible string passes over a smooth fixed pulley. It carries at one end a particle of mass 7 kg and at the other end a light, smooth pulley over which passes a light string with particles of mass 3 kg and 5 kg at its ends.



(i) On separate diagrams show the forces acting on each particle and on the movable pulley.

(ii) Find the accelerations of the three particles when the system is released from rest.

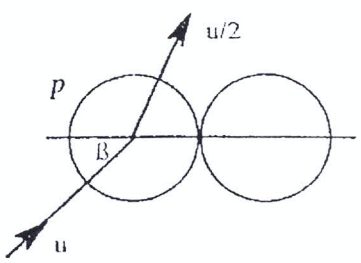
(iii) If the 3 kg mass is replaced by a mass of m kg, find the value of m if this particle does not move when the system is released from rest.

5. *(a)* Two smooth spheres of masses $2m$ and m moving in opposite directions with speeds u and $2u$, respectively, collide directly. If E_1 and E_2 are the sums of the kinetic energies of the two spheres before and after impact respectively, prove that

$$e = \sqrt{\frac{E_2}{E_1}}$$

where e is the coefficient of restitution.

(b) A smooth sphere P, moving with velocity u, impinges on an equal smooth sphere at rest, the direction of u just before impact being inclined at an angle β to the line of centres. If the speed of P after impact is $\frac{u}{2}$ and $\tan\beta = \frac{1}{2}$, show that the coefficient of restitution is also $\frac{1}{2}$.



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