1990 – 1983

LEAVING CERTIFICATE EXAMINATION, 1990

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 22 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s².
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

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1. (a) A particle is projected vertically upwards with velocity u m/s and is at a height h after t_1 and t_2 seconds respectively. Prove that

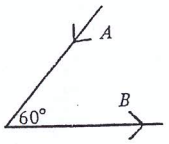
$$t_1 \cdot t_2 = \frac{2h}{g}$$

(b) A car accelerates uniformly from rest to a speed v m/s. It continues at this constant speed for t seconds and then decelerates uniformly to rest.
The average speed for the journey is $\frac{3v}{4}$.

(i) Draw a speed-time graph and hence, or otherwise, prove that the time for the journey is $2t$ seconds.

(ii) If the car-driver had observed the speed limit of $\frac{1}{2}v$, find the least time the journey would have taken, assuming the same acceleration and deceleration as in (i).

2. Two straight roads intersect at an angle of 60°.
Car A moves towards the junction with uniform speed 16 m/s, while car B moves away from the junction with uniform speed 20 m/s.



(a) Calculate the velocity of A relative to B.

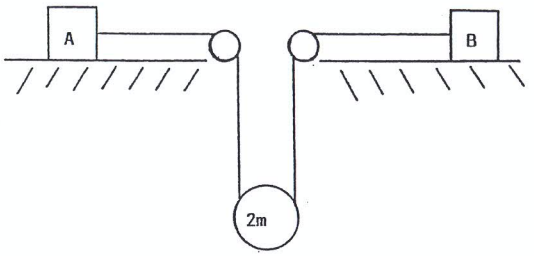
(b) If A is 450 m and B is 200 m from the intersection at a given moment, calculate the time interval in seconds until the cars
(i) are nearest to each other
(ii) are equidistant from the intersection.

3. A particle is projected from a point p, up a plane inclined at an angle $\tan^{-1}\frac{1}{6}$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope.)

(i) If the particle were to strike the inclined plane horizontally at a point q, show that $\tan\alpha = \frac{3}{19}$.

(ii) If the particle were to be projected from p with the same speed but at an angle $\tan^{-1} 3$ to the inclined plane, show that it would strike the plane at right angles at q.

4. Two blocks A and B each of mass m kg, lie at rest on horizontal rough tables. The coefficient of friction between A and the table is μ, and between B and its table is $\frac{1}{4}$. The blocks are connected by a light inextensible string which passes under a smooth movable pulley of mass $2m$ kg.



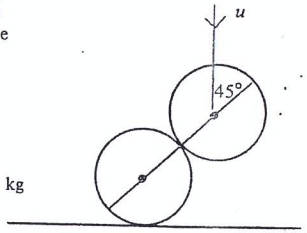
(i) Show in a diagram the forces acting on each mass when the system is released from rest.

(ii) If $\mu < \frac{3}{4}$, prove that the tension in the string is $\frac{mg(9 + 4\mu)}{16}$

(iii) Prove that A will not move if $\mu > \frac{3}{4}$.

5. State the laws governing the oblique collision of two smooth elastic spheres.

A smooth elastic sphere of mass 6 kg rests on a smooth horizontal table. A second smooth elastic sphere of mass 4 kg falls vertically on it. At the moment of impact the line of centres makes an angle of 45° with the vertical, and the velocity of the falling sphere is u. The 6 kg sphere moves horizontally after the collision.



(i) Explain why the principle of conservation of momentum may be applied horizontally.

(ii) Hence, or otherwise, prove that the speed of the 6 kg mass after impact is $\frac{u(1 + e)}{4}$ where e is the coefficient of restitution between the two spheres.

(iii) If $e = \frac{1}{3}$, prove that the loss of kinetic energy due to the impact is

$$\frac{2u^2}{3}$$

6. (a) A particle starts from rest, and moves with simple harmonic motion of period $6n$ seconds. Show that the particle moves from the position of maximum velocity to the position in which the velocity is half the maximum in n seconds.

(b) The depth of water in a harbour is assumed to rise and fall with time in simple harmonic motion. On a certain day the low tide had a height of 13 m at 12.58 p.m. and the following high tide had a height of 18 m at 6.58 p.m.

If a ship requires a depth of 16·5 m of water before it can leave the harbour, find the latest time on that day that the ship can leave the harbour.