Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2017

Marking Scheme

Applied Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. **(a)** A car passes four collinear markers *A*, *B*, *C*, and *D* while moving in a straight line with uniform acceleration. The car takes *t* seconds to travel from *A* to *B*, *t* seconds to travel from *B* to *C* and *t* seconds to travel from *C* to *D*.

If $|AB| + |CD| = k|BC|$, find the value of *k*.

$$|AB| = ut + \frac{1}{2}at^2 \qquad 5$$

$$|AC| = 2ut + \frac{1}{2}a(2t)^2 = 2ut + 2at^2$$

$$|AD| = 3ut + \frac{1}{2}a(3t)^2 = 3ut + \frac{9}{2}at^2$$

$$|BC| = ut + \frac{3}{2}at^2 \qquad 5$$

$$|CD| = ut + \frac{5}{2}at^2 \qquad 5$$

$$|AB| + |CD| = 2ut + 3at^2 \qquad 5$$

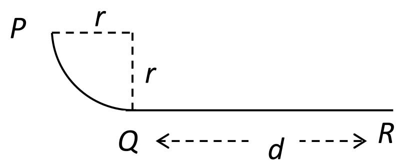
$$= 2\left\{ut + \frac{3}{2}at^2\right\}$$

$$= 2 \times |BC|$$

$$k = 2 \qquad 5$$

25

1. (b) A baggage chute has two sections, *PQ* and *QR*, as shown in the diagram.
PQ is smooth and is a quarter circle of radius *r*.
QR, of length *d*, is rough and horizontal.
The coefficient of friction between the bag and section *QR* is μ.
A bag of mass *m* kg is released from rest at *P* and comes to rest at *R*.
Find



(i) the speed of the bag at *Q* in terms of *r*
(ii) *d* in terms of μ and *r*.

The speed of the bag when it is halfway along *QR* is 7 m s^{-1}**.**

(iii) Find the value of *r*.

(i) $$\frac{1}{2}mv^2 = mgh$$ 5

$$v = \sqrt{2gr}$$ 5

(ii) QR $$v^2 = u^2 + 2as$$

$$0 = 2gr + 2ad$$

$$a = -\frac{gr}{d}$$ 5

$$F = -\mu R$$

$$m\left(-\frac{gr}{d}\right) = -\mu mg$$

$$d = \frac{r}{\mu}$$ 5

(iii) $$v^2 = u^2 + 2as$$

$$49 = 2gr + 2\left(-\frac{gr}{d}\right) \times \frac{d}{2}$$

$$= gr$$

$$r = 5 \text{ m}$$ 5

25

2. **(a)** A girl cycles at a constant speed of 5 m s^{-1} on level ground.
When her velocity is $5\vec{j}$ m s^{-1} the velocity of the wind appears to be $3u\vec{i} - 4u\vec{j}$, where u is a positive constant.
When the girl cycles with velocity $-3\vec{i} + 4\vec{j}$, the velocity of the wind appears to be $v\vec{i}$, where v is a positive constant.

Find the magnitude and direction of the velocity of the wind.

$$\vec{v}_w = \vec{v}_{wg} + \vec{v}_g$$
$$= 3u\,\vec{i} - 4u\,\vec{j} + 5\,\vec{j}$$
$$= 3u\,\vec{i} + (5 - 4u)\,\vec{j} \qquad 5$$

$$\vec{v}_w = \vec{v}_{wg} + \vec{v}_g$$
$$= v\,\vec{i} - 3\,\vec{i} + 4\,\vec{j}$$
$$= (v - 3)\,\vec{i} + 4\,\vec{j} \qquad 5$$

$$4 = 5 - 4u \Rightarrow u = \frac{1}{4} \qquad 5$$
$$\vec{v}_w = 3u\,\vec{i} + (5 - 4u)\,\vec{j}$$
$$= \frac{3}{4}\,\vec{i} + 4\,\vec{j} \qquad 5$$

$$|\vec{v}_w| = \sqrt{\left(\frac{3}{4}\right)^2 + 4^2} = 4.07 \text{ m s}^{-1}$$
$$\alpha = \tan^{-1}\left(\frac{16}{3}\right) \text{ or } 79.38^\circ \qquad 5$$

25

2 (b) A ship P is moving north at 15 km h^{-1}. A second ship Q is 10 km west of P and appears to be moving relative to P in a direction east 60° south at $15\sqrt{3}$ km h^{-1}.

(i) Find the velocity of Q.

Two minutes after the time when P and Q are closest to each other, P is east θ° north of Q.

(ii) Find the value of θ.

(iii) If Q can be seen from P for 12 minutes, find the distance between P and Q at the end of this time.

(i)

$$\vec{V}_{Q} = \vec{V}_{QP} + \vec{V}_{P}$$

$$= 15\sqrt{3}\cos 60\,\vec{i} - 15\sqrt{3}\sin 60\,\vec{j} + 15\,\vec{j}$$ 5

$$= \frac{15\sqrt{3}}{2}\vec{i} - \frac{15}{2}\vec{j}$$

$$\left|\vec{V}_{Q}\right| = 15 \text{ km h}^{-1} \qquad \text{E}\,30°\,\text{S}$$ 5

(ii)

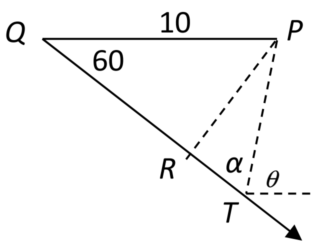
$$|PR| = 10\sin 60 = 5\sqrt{3}$$ 5

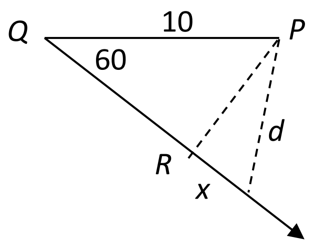
$$|RT| = 15\sqrt{3}\times\left(\frac{2}{60}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\alpha = \frac{5\sqrt{3}}{\sqrt{3}/2} = 10 \Rightarrow \alpha = 84.3°$$

$$\theta = 180 - 84.3 - 60$$

$$= 35.7°$$ 5





(iii)

$$x = 15\sqrt{3}\times\left(\frac{6}{60}\right) = 1.5\sqrt{3}$$

$$d = \sqrt{\left(1.5\sqrt{3}\right)^2 + \left(5\sqrt{3}\right)^2}$$

$$= 9.04 \text{ km}$$ 5

25

3. **(a)** A particle is projected with speed $\sqrt{\frac{9gh}{2}}$ from a point *P* on the top of a cliff of height h. It strikes the ground a horizontal distance $3h$ from *P*.

(i) Find the two possible angles of projection.

(ii) For each angle of projection find, in terms of h, the time it takes the particle to reach *P*.

(i)

$$r_{\vec{i}} = 3h$$

$$u\cos\alpha \times t = 3h$$

$$t = \frac{3h}{u\cos\alpha}$$

$$r_{\vec{j}} = -h$$

$$u\sin\alpha \times t - \tfrac{1}{2}gt^2 = -h$$

$$u\sin\alpha\left(\frac{3h}{u\cos\alpha}\right) - \tfrac{1}{2}g\left(\frac{3h}{u\cos\alpha}\right)^2 = -h$$

$$\tan^2\alpha - 3\tan\alpha = 0$$

$$\tan\alpha = 0 \quad \Rightarrow \alpha = 0^\circ$$

$$\tan\alpha = 3 \quad \Rightarrow \alpha = 71.6^\circ$$

(ii)

$$t_0 = 0 \;\text{ or }\; t_0 = \frac{3h}{u\cos\alpha}$$

$$= \frac{3h}{\sqrt{\frac{9gh}{2}}\cos 0} = \sqrt{\frac{2h}{g}}$$

$$t_3 = 0 \;\text{ or }\; t_3 = \frac{3h}{u\cos\alpha}$$

$$= \frac{3h}{\sqrt{\frac{9gh}{2}}\cos 71.6} = \sqrt{\frac{20h}{g}}$$

Marks
5
5
5
5
5

Total: 25

Note: Part **(a)(ii)** should have stated that the particle reached the ground (not *P*). Award the last 5 marks to all candidates who have work in this part.

3 (b) A plane is inclined at an angle θ (where $\theta < 45°$) to the horizontal.
A particle is projected up the plane with initial speed u m s^{-1} at an angle θ to the inclined plane.
The plane of projection is vertical and contains the line of greatest slope.

(i) Show that the range on the inclined plane is $\dfrac{2u^2 \cos 2\theta \sin\theta}{g\cos^2\theta}$.

(ii) If the particle strikes the inclined plane at right angles show that the range is $\dfrac{u^2}{g\sqrt{3}}$.

(i)

$$r_j = 0$$

$$u\sin\theta \times t - \tfrac{1}{2} g\cos\theta \times t^2 = 0$$ 5

$$t = \frac{2u\sin\theta}{g\cos\theta}$$ 5

$$r_{\bar{i}} = u\cos\theta \times t - \tfrac{1}{2} g\sin\theta \times t^2$$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g\cos\theta} - \tfrac{1}{2} g\sin\theta \times \left(\frac{2u\sin\theta}{g\cos\theta}\right)^2$$

$$= \frac{2u^2\sin\theta}{g} - \frac{2u^2\sin^3\theta}{g\cos^2\theta}$$

$$= \frac{2u^2\sin\theta}{g\cos^2\theta}\left\{\cos^2\theta - \sin^2\theta\right\}$$

$$R = \frac{2u^2\sin\theta\cos 2\theta}{g\cos^2\theta}$$ 5

(ii)

$$v_i = 0$$

$$u\cos\theta - g\sin\theta \times t = 0$$

$$t = \frac{u\cos\theta}{g\sin\theta}$$ 5

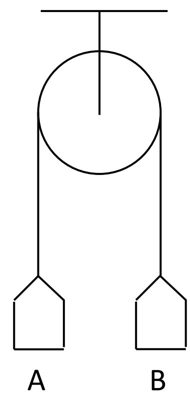
$$\frac{u\cos\theta}{g\sin\theta} = \frac{2u\sin\theta}{g\cos\theta} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$$

$$R = \frac{2u^2\sin\theta}{g\cos^2\theta}\left\{\cos^2\theta - \sin^2\theta\right\}$$

$$= \frac{2u^2}{g}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{3}{2}\right)\left\{\frac{2}{3} - \frac{1}{3}\right\} = \frac{u^2}{g\sqrt{3}}$$ 5

25

4. **(a)** Two scale pans A and B, each of mass m kg, are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. They are held at the same level, as shown in the diagram. A mass of $3m$ kg is now placed on A.



The system is released from rest.

Find

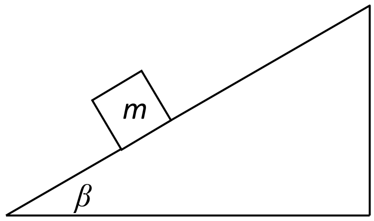
(i) the tension in the string in terms of m

(ii) how far B has risen when it reaches a speed of 0·4 m s^{-1}

(iii) the reaction on the $3m$ kg mass in terms of m.

(i)	$4mg - T = 4ma$	5
	$T - mg = ma$	5
	$a = \dfrac{3g}{5}$	
	$T = \dfrac{8mg}{5}$	5
(ii)	$v^2 = u^2 + 2as$	
	$0.16 = 0 + 2 \times \dfrac{3g}{5} \times h$	
	$h = 0.0136$ m	5
(iii)	$3mg - R = 3ma$	
	$3mg - R = 3m\left(\dfrac{3g}{5}\right)$	
	$R = \dfrac{6mg}{5}$	5
		25

4. (b) A smooth wedge, of mass $4m$ and slope β, rests on a smooth horizontal surface.
A particle of mass m is placed on the smooth inclined face of the wedge.
The system is released from rest.



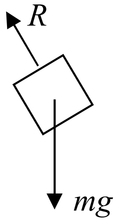
(i) Show, on separate diagrams, the forces acting on the wedge and on the particle.

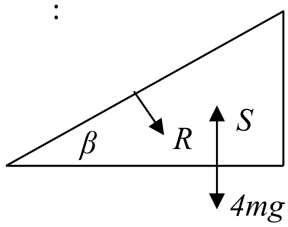
The particle moves with acceleration p relative to the wedge and the wedge moves with acceleration $kp\cos\beta$.

(ii) Find the value of k.

(iii) Show that $p = \dfrac{49\sin\beta}{4+\sin^2\beta}$.

(i)





5,5

(ii)

$$mg\cos\beta - R = mkp\cos\beta\sin\beta$$

5

$$mg\sin\beta = m\left(p - kp\cos^2\beta\right)$$

$$R\sin\beta = 4mkp\cos\beta$$

$$R\sin\beta = 4mkp\cos\beta$$

$$\{mg\cos\beta - mkp\cos\beta\sin\beta\}\sin\beta = 4mkp\cos\beta$$

$$mg\sin\beta - mkp\sin^2\beta = 4mkp$$

$$m\left(p - kp\cos^2\beta\right) - mkp\sin^2\beta = 4mkp$$

$$p - kp\left(\cos^2\beta + \sin^2\beta\right) = 4kp$$

$$k = \frac{1}{5}$$

5

(iii)

$$mg\sin\beta = m\left(p - kp\cos^2\beta\right)$$

$$5g\sin\beta = 5p - p\cos^2\beta$$

$$= p\left(5 - 1 + \sin^2\beta\right)$$

$$p = \frac{49\sin\beta}{4+\sin^2\beta}$$

5

25

5. **(a)** A small smooth sphere A, of mass 1·5 kg, moving with speed 6 m s^{-1}, collides directly with a small smooth sphere B, of mass *m* kg, which is at rest.
After the collision the spheres move in opposite directions with speeds *v* and 2*v*, respectively.
80% of the kinetic energy lost by A as a result of the collision is transferred to B.
The coefficient of restitution between the spheres is *e*.

Find **(i)** the value of *v*
(ii) the value of *e*.

(i) PCM $1.5(6) + m(0) = 1.5(-v) + m(2v)$ **5**

$$m = \frac{9 + 1.5v}{2v}$$

$$0.8\left\{\tfrac{1}{2}(1.5)(6^2) - \tfrac{1}{2}(1.5)v^2\right\} = \tfrac{1}{2}m(2v)^2$$

$$21.6 - 0.6v^2 = 9v + 1.5v^2$$

$$2.1v^2 + 9v - 21.6 = 0$$

$$v = \frac{12}{7}\ \text{m s}^{-1}$$ **5**

(ii) NEL $v_1 - v_2 = -e(6 - 0)$ **5**

$$-v - 2v = -6e$$

$$e = \frac{v}{2} = \frac{6}{7}$$ **5**

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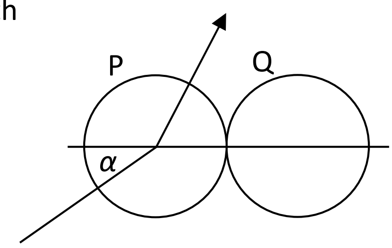
5. (b) A small smooth sphere P, of mass $3m$, collides obliquely with a small smooth sphere Q, of mass $7m$, which is at rest.



Before the collision the velocity of P makes an angle α with the line joining the centres of the spheres.
After the collision the speed of Q is v.
The coefficient of restitution between the spheres is $\frac{2}{7}$.

(i) Find, in terms of v and α, the **speed** of P before the collision.

(ii) If $\alpha = 30°$ find the angle through which the direction of motion of P is deflected as a result of the collision.

P	$3m$	$u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$	$v_1\,\vec{i} + u\sin\alpha\,\vec{j}$
Q	$7m$	$0\,\vec{i} + 0\,\vec{j}$	$v\,\vec{i} + 0\,\vec{j}$

(i) PCM $\quad 3m(u\cos\alpha) + 7m(0) = 3m(v_1) + 7m(v)$ 5

NEL $\quad v_1 - v = -\frac{2}{7}(u\cos\alpha - 0)$ 5

$$v = \frac{27u\cos\alpha}{70}$$ 5

$$u = \frac{70v}{27\cos\alpha}$$ 5

(ii)

$$v_1 = \frac{u\cos\alpha}{10}$$ 5

$$\tan\beta = \frac{u\sin\alpha}{v_1}$$

$$= \frac{10u\sin\alpha}{u\cos\alpha} = 10\tan 30$$

$$= \frac{10}{\sqrt{3}}$$

$$\beta = 80.17$$

angle $= 80.17 - 30 = 50.17°$ 5

30

6. **(a)** Two particles moving with simple harmonic motion pass through their centres of oscillation at the same instant.
They next reach their greatest distances from their centres of oscillation
\` after 2 seconds and 3 seconds respectively, having been at the same distance from their centres of oscillation after 1 second.

Find the ratio of their amplitudes.

$$\frac{2\pi}{\omega_1} = 8 \;\Rightarrow\; \omega_1 = \frac{\pi}{4}$$ 5

$$x = A_1 \sin\frac{\pi}{4} = \frac{A_1}{\sqrt{2}}$$ 5

$$\frac{2\pi}{\omega_2} = 12 \;\Rightarrow\; \omega_2 = \frac{\pi}{6}$$ 5

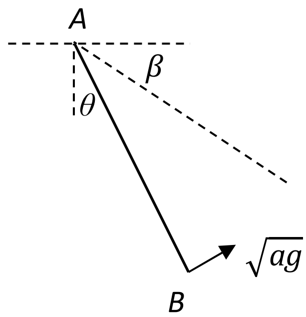
$$x = A_2 \sin\frac{\pi}{6} = \frac{A_2}{2}$$ 5

$$\frac{A_1}{\sqrt{2}} = \frac{A_2}{2}$$

$$\frac{A_1}{A_2} = \frac{\sqrt{2}}{2} \quad \text{or} \quad \frac{1}{\sqrt{2}}$$ 5

25

6. (b) One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The string makes an angle θ with the vertical.
The particle is held in equilibrium with the string taut and $\cos\theta = \frac{2}{3}$. The particle is then projected with speed $\sqrt{ag}$, in the direction perpendicular to AB, as show in the diagram.
In the subsequent motion the string remains taut.
When AB makes an angle β below the horizontal, the speed of the particle is v and the tension in the string is T.



(i) Show that $v^2 = 3ag(2\sin\beta - 1)$.
(ii) Find the minimum value and the maximum value of T.

(i) $\frac{1}{2}m(ag) = \frac{1}{2}mv^2 + mg\{3a\cos\theta - 3a\sin\beta\}$ — 5, 5

$$ag = v^2 + 6ag\cos\theta - 6ag\sin\beta$$

$$ag = v^2 + 4ag - 6ag\sin\beta$$

$v^2 = 3ag(2\sin\beta - 1)$ — 5

(ii) T_{max} when $\beta = \dfrac{\pi}{2}$

$$v^2 = 3ag(2-1) = 3ag$$

$$T - mg = \frac{mv^2}{3a}$$

$$T_{max} - mg = \frac{m(3ag)}{3a}$$

$T_{max} = 2mg$ — 5

T_{min} when $v = 0$

$$v^2 = 3ag(2\sin\beta - 1)$$

$$0 = 3ag(2\sin\beta - 1) \Rightarrow \beta = 30^\circ$$

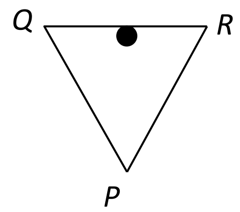
$$T - mg\sin\beta = \frac{mv^2}{3a}$$

$$T_{min} - mg\left(\frac{1}{2}\right) = 0$$

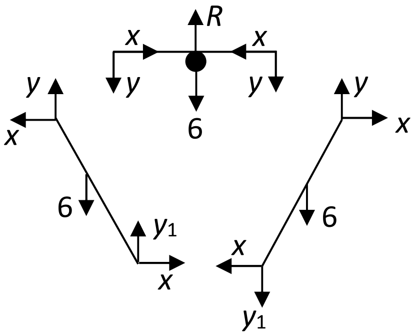
$T_{min} = \frac{1}{2}mg$ — 5

25

7. **(a)** Three equal uniform rods, QP, QR, RP, each 30 cm long and of weight 6 N, are freely jointed at P, Q and R to form a triangle. The triangle is placed over a smooth peg at the midpoint of QR, so that P is below QR.



Find the reaction at Q and the reaction at P.



$$y_1 = 0$$ 5

$$y + y_1 = 6$$

$$\Rightarrow y = 6$$ 5

$$x(0.3\sin 60) + 6(0.15\cos 60) = 6(0.3\cos 60)$$ 5

$$x\tan 60 + 3 = 6$$

$$x = \sqrt{3}$$ 5

$$R_Q = \sqrt{\left(\sqrt{3}\right)^2 + 6^2}$$

$$= \sqrt{39}$$

$$= 6.24 \text{ N}$$

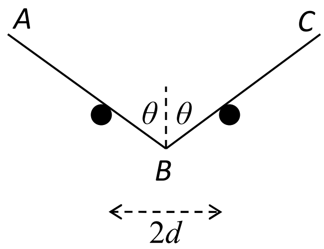
$$R_P = \sqrt{3} \text{ N}$$ 5 **25**

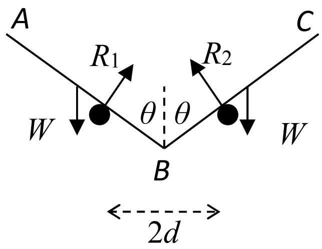
7. (b) Two equal uniform rods, *AB* and *BC*, of length $2a$ and weight W, are freely jointed at *B*.

They rest in equilibrium on two smooth pegs at the same horizontal level which are $2d$ apart.

Each rod is inclined at $\theta°$ to the vertical. *B* is below *A* and *C*.

Prove $d = a\sin^3\theta$.





$$R_1\cos\theta = R_2\cos\theta$$ 5

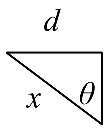
$$R_1 = R_2$$

$$R_1\sin\theta + R_2\sin\theta = W + W$$ 5

$$2R_1\sin\theta = 2W$$

$$R_1\sin\theta = W$$

$$R_1 = \frac{W}{\sin\theta}$$ 5



$$R_1(x) = W(a\sin\theta)$$ 5

$$\left(\frac{W}{\sin\theta}\right)\times\left(\frac{d}{\sin\theta}\right) = W\times a\sin\theta$$

$$d = a\sin^3\theta$$ 5

25

[15]

8. (a) Prove that the moment of inertia of a uniform rod, of mass m and length $2l$ about an axis through its centre, perpendicular to its plane, is $\frac{1}{3}ml^2$.

Let M = mass per unit length

$$\text{mass of element} = \text{M}\{\text{d}x\}$$

$$\text{moment of inertia of the element} = \text{M}\{\text{d}x\}x^2 \qquad 5$$

$$\text{moment of inertia of the rod} = \text{M}\int_{-\ell}^{\ell} x^2\,\text{d}x \qquad 5$$

$$= \text{M}\left[\frac{x^3}{3}\right]_{-\ell}^{\ell} \qquad 5$$

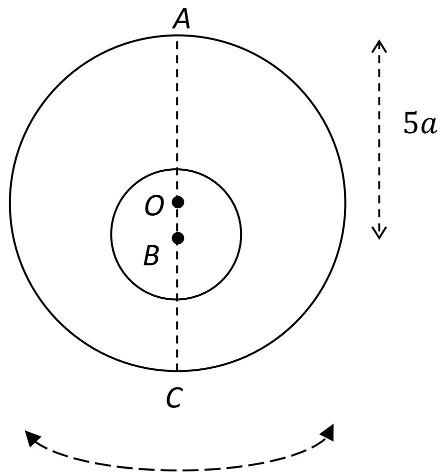
$$= \tfrac{2}{3}\text{M}\ell^3$$

$$= \tfrac{1}{3}m\,\ell^2 \qquad 5$$

20

8. (b) A uniform circular disc has mass $4m$, centre O and radius $4a$.
A circular hole of radius $2a$ is made in the disc.
The centre of the hole is at the point B on the diameter AC, where $|AB| = 5a$,
as shown in the diagram.
The resulting lamina oscillates about a fixed smooth horizontal axis which passes through A and is perpendicular to the plane of the lamina.

(i) Find the moment of inertia of the lamina about the axis of rotation.



The lamina is hanging at rest with C vertically below A when it is given an angular velocity $\sqrt{\dfrac{11g}{23a}}$.
The lamina turns through an angle θ before it comes to instantaneous rest.

(ii) Show that the distance of A from the centre of gravity of the lamina is $\frac{11a}{3}$.

(iii) Find the value of $\boldsymbol{\theta}$.

(i) $\text{mass of disc removed} = \frac{\text{mass}}{\text{area}} \times \pi(2a)^2$

$$= \frac{4m}{\pi(4a)^2} \times \pi(2a)^2 = m$$ 5

$$I = \left\{\tfrac{1}{2}(4m)(4a)^2 + (4m)(4a)^2\right\}$$ 5

$$-\left\{\tfrac{1}{2}(m)(2a)^2 + (m)(5a)^2\right\}$$ 5

$$= 69ma^2$$ 5

(ii) $3m(d) = 4m(4a) - m(5a)$

$$3d = 16a - 5a$$

$$d = \frac{11a}{3}$$ 5

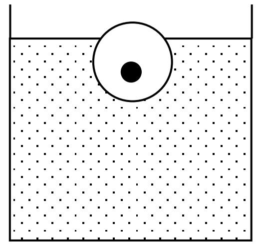
(iii) $3mg\left(\frac{11a}{3} + \frac{11a}{3}\cos\alpha\right) = \frac{1}{2}\left(69ma^2\right)\left(\frac{11g}{23a}\right)$

$$1 + \cos\alpha = \frac{3}{2}$$

$$\alpha = 60$$

$$\theta = 120^\circ$$ 5 30

9. **(a)** A spherical piece of ice of radius 10 cm has a piece of iron embedded in it and floats, in water, with 95% of its volume immersed.



The density of the iron is 7800 kg m^{-3} and the density of ice is 918 kg m^{-3}.

Find the volume of the iron.

[Density of water = 1000 kg m^{-3}]

$$B = 1000 \times \left\{\tfrac{4}{3}\pi(0.1)^3\right\} \times g \times 0.95$$ **5**

$$W = 7800Vg + 918 \times \left\{\tfrac{4}{3}\pi(0.1)^3 - V\right\} \times g$$ **5**

$$B = W$$

$$950 \times \left\{\tfrac{4}{3}\pi(0.1)^3\right\} \times g = 7800Vg + 918 \times \left\{\tfrac{4}{3}\pi(0.1)^3 - V\right\} \times g$$ **5**

$$1.2666\pi = 7800V + 1.224\pi - 918V$$

$$V = \frac{0.0426\pi}{6882}$$

$$= 1.94 \times 10^{-5} \text{ m}^3$$ **5** **20**

9 **(b)** A solid iron cylinder of diameter 25 cm and height 10 cm is placed upright in an empty cylindrical tank of diameter 30 cm.
Mercury is now poured into the tank until the iron begins to float.
The density of iron is 7800 kg m^{-3} and the density of mercury is 13 600 kg m^{-3}.

(i) Calculate the depth of the mercury.

Oil, of density 1030 kg m^{-3}, is now poured on top of the mercury until the upper face of the iron cylinder is just level with the surface of the oil.

(ii) Find the volume of oil required.

(i)

$$B = 13600 \times \{\pi(0.125)^2 h\} \times g \qquad 5$$

$$W = 7800 \times \{\pi(0.125)^2 (0.1)\} \times g \qquad 5$$

$$B = W$$

$$13600 \times \{\pi(0.125)^2 h\} \times g = 7800 \times \{\pi(0.125)^2 (0.1)\} \times g \qquad 5$$

$$h = \frac{39}{680} \quad \text{or} \quad 0.057 \text{ m} \qquad 5$$

(ii)

$$B = 13600 \times \{\pi(0.125)^2 (0.1 - h)\} \times g$$

$$+ 1030 \times \{\pi(0.125)^2 h\} \times g \qquad 5$$

$$W = 7800 \times \{\pi(0.125)^2 (0.1)\} \times g$$

$$B = W$$

$$1360(0.1 - h) + 103h = 78$$

$$h = \frac{58}{1257} \quad \text{or} \quad 0.046 \text{ m}$$

$$V = \pi(0.15)^2 h - \pi(0.125)^2 h$$

$$= 0.0216 \times 0.046$$

$$= 9.94 \times 10^{-4} \text{ m}^3 \qquad 5 \qquad 30$$

10. **(a)** A particle starts from rest and moves in a straight line with acceleration $(25 - 10v)$ m s^{-2}, where v is the speed of the particle.

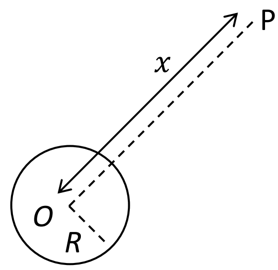
(i) After time t, find v in terms of t. (Note: $\int \frac{dx}{a+bx} = \frac{1}{b} ln|a + bx| + c$).

(ii) Find the time taken to acquire a speed of 2·25 m s^{-1} and find the distance travelled in this time.

(i)	$\int \frac{dv}{25\text{-}10v} = \int dt$	5
	$\left[-\frac{1}{10}\ln(25-10v)\right]_0^v = [t]_0^t$	5
	$-\frac{1}{10}\ln(25-10v) + \frac{1}{10}\ln 25 = t$	5
	$\ln\frac{25}{25-10v} = 10t$	
	$v = 2.5\left(1 - e^{-10t}\right)$	5
(ii)	$\ln\frac{25}{25-10v} = 10t$	
	$10t = \ln\frac{25}{25-22.5}$	5
	$t = \frac{1}{10}\ln 10 = 0.23$ s	
	$\int ds = 2.5\int \left(1 - e^{-10t}\right)dt$	
	$s = 2.5\left[t + \frac{1}{10}e^{-10t}\right]_0^{0.23}$	
	$= 0.35$ m	5

30

10 **(b)** A spacecraft P of mass *m* moves in a straight line towards *O*, the centre of the earth.
The radius of the earth is *R*.
When P is a distance x from *O*, the force exerted by the earth on P is directed towards *O* and has magnitude $\frac{k}{x^2}$, where k is a constant.



(i) Show that $k = mgR^2$.

P starts from rest when its distance from *O* is 5*R*.

(ii) Find, in terms of *R*, the speed of P as it hits the surface of the earth, given that air resistance can be ignored.

(i)

$$mg = \frac{k}{R^2}$$

$$k = mgR^2$$ 5

(ii)

$$F = -\frac{k}{x^2} = -\frac{mgR^2}{x^2}$$ 5

$$mv\frac{dv}{dx} = -\frac{mgR^2}{x^2}$$

$$\int v\,dv = -gR^2\int x^{-2}\,dx$$ 5

$$\left[\tfrac{1}{2}v^2\right]_0^v = \left[\frac{gR^2}{x}\right]_{5R}^{R}$$

$$\tfrac{1}{2}v^2 = gR - \frac{gR}{5}$$

$$v^2 = \frac{8gR}{5}$$

$$v = \sqrt{\frac{8gR}{5}} \quad \text{or} \quad 3.96\sqrt{R}$$ 5 | 20