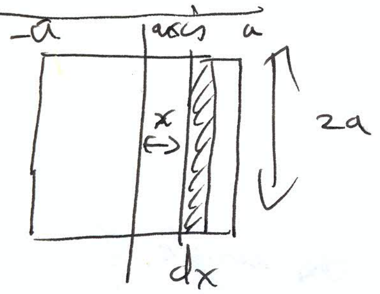
APM

HLC 1995 Q8



MICRO $dm = \rho\, dA$

$dm = \rho(2a)\, dx$

macro

$M = \rho (2a)^2$

$\rho = \frac{M}{(2a)^2}$

$$dI = dm\, x^2 \quad [5]$$

$$\Rightarrow I = \int_{-a}^{a} \rho (2a)\, x^2\, dx \quad [5]$$

$$\Rightarrow I = \rho\, 2a \int_{-a}^{a} x^2\, dx$$

$$I = 2a\rho\, \frac{x^3}{3}\Big|_{-a}^{a} \quad [5]$$

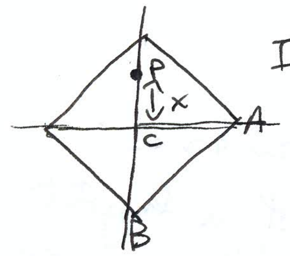
$$I = 2a\rho\left(\left[\frac{a^3}{3}\right] - \left[-\frac{a^3}{3}\right]\right)$$

$$I = \frac{4a^4\rho}{3}$$

$$I = \frac{4a^4}{3} \cdot \frac{M}{(2a)^2}$$

$$I = \frac{1}{3} M a^2 \quad [5]$$

(f)



$$I_P = I_C + m x^2 \quad (\parallel \text{ axes})$$

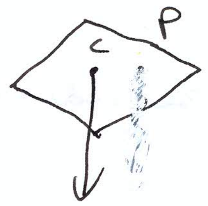
$$= [I_A + I_B] + m x^2 \quad (\perp \text{ axes})$$

$$= \frac{1}{3} m a^2 + \frac{1}{3} m a^2 + m x^2$$

$$\boxed{I_P = \frac{2}{3} m a^2 + m x^2} \quad [5]$$

$$T = 2\pi \sqrt{\frac{I}{Mgh}} \quad [5]$$

Find h : $h = |PC| = x$



M = mass total $= m$ [5]

$$\therefore T = 2\pi \sqrt{\frac{\frac{2}{3} m a^2 + m x^2}{m g x}} \quad [5]$$

$$T^2 = (2\pi)^2 \left(\frac{\frac{2}{3} a^2 + x^2}{g x}\right)$$

$$\frac{dT^2}{dx} = (2\pi)^2 \cdot \left(\frac{gx[2x] - (\frac{2}{3}a^2 + x^2)[g]}{(gx)^2}\right)$$

$$\left[\frac{dT^2}{dx} = \frac{dT^2}{dT}\frac{dT}{dx} \quad \frac{dT^2}{dx} = 0 \Leftrightarrow \frac{dT}{dx} = 0\right]$$

$$\therefore \frac{2gx^2 - (\frac{2}{3}a^2 + x^2) g}{(gx)^2} = 0 \quad [5]$$

$$\Rightarrow 2gx^2 - \frac{2}{3}a^2 g - g x^2 = 0$$

$$\Rightarrow x^2 = \frac{2}{3} a^2$$

$$\Rightarrow \boxed{3x^2 = 2a^2} \quad [5]$$

qed