Eg ② (1996 Q6)

(a) A body of mass 10 kg moves with simple harmonic motion. At a displacement of 0.8 m from the centre of oscillation, the velocity and acceleration of the body are 2 m/s and 20 m/s² respectively.

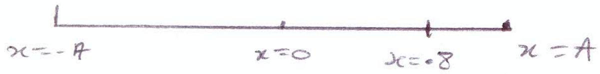
Find
- *(i)* the number of oscillations per second
- *(ii)* the amplitude of motion
- *(iii)* the maximum acceleration and hence show that the force to overcome the inertia of the body at the extremity of the oscillation is 223.6N.

(b) A light perfectly elastic string of natural length a and elastic constant k is fastened at one end p to a fixed point of a smooth horizontal table, and a particle of mass m is attached to the other end. The particle is held on the table at a distance $2a$ from p and then released.

Prove
- *(i)* that the particle executes simple harmonic motion while the string is taut
- *(ii)* that the particle reaches p after

$$\left(\frac{\pi}{2}+1\right)\sqrt{\frac{m}{k}} \text{ seconds.}$$

(a)



$|a| = 20$
$v = 2$

(i) $|a| = \omega^2 x \Rightarrow 20 = \omega^2(\cdot 8)$
(Find ω) $\Rightarrow \frac{20}{\cdot 8} = \omega^2$
$\Rightarrow 25 = \omega^2$
$\Rightarrow \underline{5 = \omega}$

Time for one osc. $= T = \frac{2\pi}{\omega}$

1 osc. $\Longleftrightarrow \frac{2\pi}{5}$ secs

$\frac{5}{2\pi}$ osc $\Longleftrightarrow$ 1 sec

$\frac{5}{2\pi}$ osc per sec

(ii) Find A $\quad v^2 = \omega^2(A^2 - x^2)$
$\Rightarrow 4 = 25(A^2 - (\cdot 8)^2)$
$\Rightarrow \frac{4}{25} = A^2 - \cdot 64$
$\Rightarrow \cdot 16 + \cdot 64 = A^2$
$\Rightarrow \cdot 8 = A^2$
$\Rightarrow A = \sqrt{\cdot 8}$ metres

(iii) Max accel is at extremity. $x = A$

$|a|_{max} = \omega^2 A = 25\sqrt{\cdot 8}$

N II $\Rightarrow F = ma$
$\Rightarrow F_{max} = m\omega^2 A$

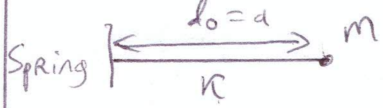
$\Rightarrow F_{max}$ to overcome inertia is

$|F_{max}| = 10(25)\sqrt{\cdot 8}$ N

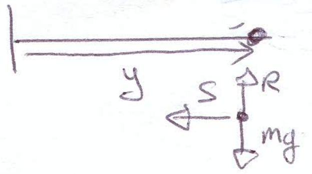
$F = 223 \cdot 6$ N

Velocity at q $x = 0 \therefore v^2 = \omega^2(A^2 - 0^2)$
$v = \omega A$
$\Rightarrow v = \sqrt{\frac{K}{m}}\, a$

(b)



(i) First let y be position of equil from wall



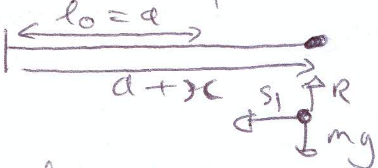
Hooke: $S = K(y - a)$

Here accel $= f = 0$, at equil.

N II $\Sigma F = mf \Rightarrow -K(y-a) = m(0)$
$\Rightarrow \boxed{y = a}$

Equil position is a from wall

NEXT Examine forces at $a + x$ from wall.

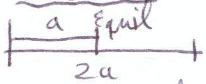


Hooke $\Rightarrow |S_1| = -K(a + x - a) = -Kx$.

N II $\Rightarrow \Sigma F = mf \Rightarrow -Kx = mf$
$\Rightarrow \boxed{f = -\frac{K}{m}x}$

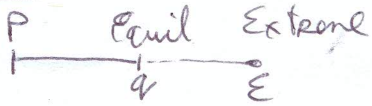
SHM about $x = 0$ with $\omega = \sqrt{\frac{K}{m}}$

(ii) First A : $v = 0$ at 2a from wall



$\Rightarrow \boxed{A = a}$

Time to reach p :



$t_{Ep} = t_{Eq} + t_{qp}$

$t_{Eq} = \frac{1}{4}T = \frac{1}{4}\left(\frac{2\pi}{\sqrt{\frac{K}{m}}}\right) = \frac{\pi}{2}\frac{\sqrt{m}}{\sqrt{K}}$.

$t_{qp} = \frac{\text{dist}(pq)}{\text{vel at } q} = \frac{a}{\sqrt{\frac{K}{m}}\, a} = \sqrt{\frac{m}{K}}$

$\therefore$ Total time $= \frac{\pi}{2}\sqrt{\frac{m}{K}} + \sqrt{\frac{m}{K}} = \sqrt{\frac{m}{K}}\left(1 + \frac{\pi}{2}\right)$ qed