

(H, 71, 05) Def<sup>n</sup> SHM:

$$\text{accel} = -\omega^2 x$$

$$\text{but accel} = \frac{dv}{dt} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d^2x}{dt^2}$$

$$\Rightarrow \left[ \frac{d^2x}{dt^2} = -\omega^2 x \right] \quad *$$

Show  $x = a \cos \omega t$  a Sol<sup>n</sup>:

$$\frac{dx}{dt} = a\omega(-\sin \omega t)$$

$$\frac{d^2x}{dt^2} = a\omega^2(\cos \omega t) \\ = -\omega^2(a \cos \omega t)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow x = a \cos \omega t \text{ satisfies the D.E. } *$$

$$\text{When } v=4, x=1 \quad v^2 = \omega^2(A^2 - x^2) \Rightarrow 4^2 = \omega^2(A^2 - 1^2) \\ \Rightarrow 16 = \omega^2(A^2 - 1) \quad (1)$$

$$\text{When } v=2, x=2 \quad v^2 = \omega^2(A^2 - x^2) \Rightarrow 2^2 = \omega^2(A^2 - 2^2) \\ \Rightarrow 4 = \omega^2(A^2 - 4) \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{16}{4} = \frac{A^2 - 1}{A^2 - 4} \Rightarrow 16(A^2 - 4) = 4(A^2 - 1) \\ \Rightarrow 16A^2 - 64 = 4A^2 - 4 \\ \Rightarrow 12A^2 = 60 \\ \Rightarrow A^2 = 5 \\ \Rightarrow \boxed{A = \pm\sqrt{5}}$$

$$\text{Find } \omega: (1) \Rightarrow 16 = \omega^2((\sqrt{5})^2 - 1) \\ \Rightarrow 16 = \omega^2(5 - 1) \Rightarrow \omega^2 = 4 \Rightarrow \boxed{\omega = 2}$$

$$\text{Find } T: \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = \pi \text{ s}}$$

Find time to reach position where  $v=2 \text{ m s}^{-1}$ .

Step I: Find distance <sub>from centre</sub> where  $v=2 \text{ m s}^{-1}$ .

$$v^2 = \omega^2(A^2 - x^2) \Rightarrow 2^2 = 2^2((\sqrt{5})^2 - x^2) \Rightarrow 1 = 5 - x^2 \\ \Rightarrow \boxed{x = \pm 2 \text{ m}}$$

Step II: Find time to go from centre to point  $x=2 \text{ m}$  away

$$x = A \sin \omega t \Rightarrow 2 = \sqrt{5} \sin 2t$$

$$\Rightarrow \frac{2}{\sqrt{5}} = \sin 2t$$

$$\Rightarrow \frac{1}{2} \sin^{-1} \frac{2}{\sqrt{5}} = t$$

$$\Rightarrow \boxed{t = 0.554 \text{ s} = t}$$

Careful of radians