2002 – Projectiles Question

3. **(a)** A particle is projected from a point on the horizontal ground with a speed of 39.2 m/s inclined at an angle α to the horizontal ground. The particle is at a height of 14.7 m above the horizontal ground at times t_1 and t_2 seconds, respectively.

(i) Show that $t_2 - t_1 = \sqrt{64\sin^2\alpha - 12}$.

(ii) Find the value of α for which $t_2 - t_1 = \sqrt{20}$.

(b) A particle is projected with velocity u m/s at an angle θ **to the horizontal**, up a plane inclined at an angle β to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle strikes the plane at right angles.

(i) Show that $2\tan\beta\tan(\theta - \beta) = 1$.

(ii) Hence, or otherwise, show that if $\theta = 2\beta$, the range of the particle up the inclined plane is $\dfrac{u^2}{g\sqrt{3}}$.

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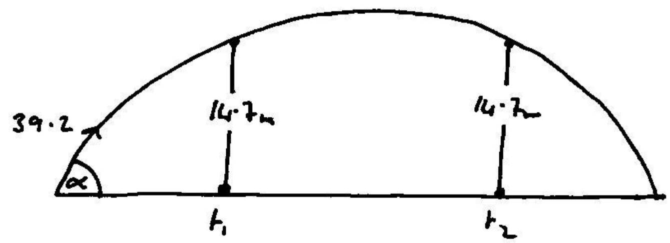
Q3/ (a)



(i) FIND TIMES WHEN $S_y = 14 \cdot 7m$

$u_y = 39 \cdot 2 \sin\alpha$
$v_y = -$
$a_y = -g$
$S_y = 14 \cdot 7m$
$t = ?$

$$S = ut + \tfrac{1}{2}at^2$$

$$14 \cdot 7 = (39 \cdot 2 \sin\alpha)t + \tfrac{1}{2}(-9 \cdot 8)(t)^2$$

$$14 \cdot 7 = 39 \cdot 2 \sin\alpha t - 4 \cdot 9t^2$$

$$4 \cdot 9t^2 - 39 \cdot 2 \sin\alpha\, t + 14 \cdot 7 = 0$$

USE $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ TO SOLVE THE QUADRATIC.

$a = 4 \cdot 9$, $b = -39 \cdot 2 \sin\alpha$, $c = 14 \cdot 7$

SO

$$t = \frac{-(-39 \cdot 2 \sin\alpha) \pm \sqrt{(-39 \cdot 2 \sin\alpha)^2 - 4(4 \cdot 9)(14 \cdot 7)}}{2(4 \cdot 9)}$$

$$t = \frac{39 \cdot 2 \sin\alpha \pm \sqrt{1536 \cdot 64 \sin^2\alpha - 288 \cdot 12}}{9 \cdot 8}$$

SO,

$$t_1 = \frac{39 \cdot 2 \sin\alpha - \sqrt{1536 \cdot 64 \sin^2\alpha - 288 \cdot 12}}{9 \cdot 8}$$

AND

$$t_2 = \frac{39 \cdot 2 \sin\alpha + \sqrt{1536 \cdot 64 \sin^2\alpha - 288 \cdot 12}}{9 \cdot 8}$$

$\longrightarrow$

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Q3/ (a) $t_2 - t_1 \Rightarrow \left[\dfrac{39.2\sin\alpha + \sqrt{1536.64\sin^2\alpha - 288.12}}{9.8}\right] - \left[\dfrac{39.2\sin\alpha - \sqrt{1536.64\sin^2\alpha - 288.12}}{9.8}\right]$

$$\Rightarrow t_2 - t_1 = \frac{\cancel{39.2\sin\alpha} + \sqrt{1536.64\sin^2\alpha - 288.12} - \cancel{39.2\sin\alpha} + \sqrt{1536.64\sin^2\alpha - 288.12}}{9.8}$$

$$t_2 - t_1 = \frac{2\sqrt{1536.64\sin^2\alpha - 288.12}}{9.8}$$

$$\Rightarrow t_2 - t_1 = \frac{\sqrt{1536.64\sin^2\alpha - 288.12}}{4.9}$$

& $4.9 = \sqrt{24.01}$, SO TAKING THIS OUT OF THE TOP

$$\Rightarrow t_2 - t_1 = \frac{\cancel{\sqrt{24.01}}\sqrt{64\sin^2\alpha - 12}}{\cancel{4.9}}$$

So, $t_2 - t_1 = \sqrt{64\sin^2\alpha - 12}$ Q

(ii) $t_2 - t = \sqrt{20}$

So,

$\sqrt{64\sin^2\alpha - 12} = \sqrt{20}$ SQUARE BOTH SIDES

$$64\sin^2\alpha - 12 = 20$$

$$64\sin^2\alpha = 32$$

$$\sin^2\alpha = \frac{1}{2}$$

$$\sin\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

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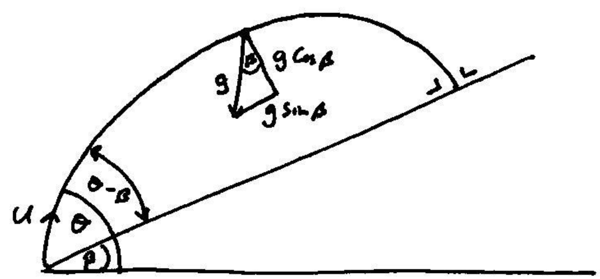
Q3/ (b)



LANDS AT RIGHT ANGLES SO, $V_x = 0_{m/s}$ WHEN $S_y = 0_m$

(i) FIND TIME WHEN $S_y = 0$:

$U_y = u \sin(\theta-\beta)$
$V_y = -$
$a_y = -g\cos\beta$
$S_y = 0$
$t = ?$

$$S = ut + \tfrac{1}{2}at^2$$

$$0 = [u\sin(\theta-\beta)]t + \tfrac{1}{2}(-g\cos\beta)t^2$$

$$0 = u\sin(\theta-\beta) - \frac{g\cos\beta}{2}t$$

$$\boxed{t = \frac{2u\sin(\theta-\beta)}{g\cos\beta}} = \text{Time of flight.}$$

$V_x = 0$ ON LANDING SO:

$U_x = u\cos(\theta-\beta)$
$V_x = 0$
$a_x = -g\sin\beta$
$S_x = -$
$t = \dfrac{2u\sin(\theta-\beta)}{g\cos\beta}$

$$V = u + at$$

$$0 = u\cos(\theta-\beta) - g\sin\beta\left[\frac{2u\sin(\theta-\beta)}{g\cos\beta}\right]$$

$$0 = \cancel{u}\cos(\theta-\beta) - \sin\beta\left[\frac{2\cancel{u}\sin(\theta-\beta)}{\cos\beta}\right]$$

$$\Rightarrow \frac{2\sin\beta\sin(\theta-\beta)}{\cos\beta} = \cos(\theta-\beta)$$

$$\text{so} \quad \frac{2\sin\beta\sin(\theta-\beta)}{\cos\beta\cos(\theta-\beta)} = 1$$

$$\Rightarrow \underline{\underline{2\tan\beta\tan(\theta-\beta) = 1}}$$

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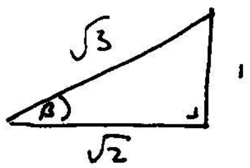
Q3/(b)(ii) $\theta = 2\beta$

so, $2 \tan\beta \tan(\theta - \beta) = 1$

$\Rightarrow 2 \tan\beta \tan\beta = 1$

$2 \tan^2\beta = 1$

$\tan^2\beta = \frac{1}{2} \Rightarrow \tan\beta = \frac{1}{\sqrt{2}}$



$\sin\beta = \frac{1}{\sqrt{3}}, \quad \cos\beta = \frac{\sqrt{2}}{\sqrt{3}}$

So TIME OF FLIGHT $= \frac{2u\sin(\theta-\beta)}{g\cos\beta} = \frac{2u\sin\beta}{g\cos\beta} = \frac{2u\tan\beta}{g} = \frac{2u}{g\sqrt{2}}$

FIND RANGE:

$u_x = u\cos(\theta-\beta) = u\cos\beta = \frac{u\sqrt{2}}{\sqrt{3}}$

$v_x = -$

$a_x = -g\sin\beta = \frac{-g}{\sqrt{3}}$

$S_x = ?$

$t = \frac{2u}{g\sqrt{2}}$

$S = ut + \frac{1}{2}at^2$

$S = \left(\frac{u\sqrt{2}}{\sqrt{3}}\right)\left(\frac{2u}{g\sqrt{2}}\right) + \frac{1}{2}\left(\frac{-g}{\sqrt{3}}\right)\left(\frac{2u}{g\sqrt{2}}\right)^2$

$S_x = \frac{2u^2}{g\sqrt{3}} - \frac{u^2}{g\sqrt{3}}$

so,

$$S_x = \frac{u^2}{g\sqrt{3}}$$

□