1995 HLC Q10

(a) $\frac{dy}{dx} = \frac{1}{y}\left(\frac{4}{1+x^2}\right)$

$\Rightarrow \int y\,dy = \int \frac{4}{1+x^2}\,dx$

$\Rightarrow \frac{y^2}{2} = 4\int \frac{1}{1+x^2}\,dx + c$

$\Rightarrow \frac{y^2}{2} = 4\ \frac{1}{1}\tan^{-1}\frac{x}{1} + c$

// $x=0,\ y=1 \Rightarrow$.

$\frac{1^2}{2} = 4\tan^{-1}(0) + c$

$\Rightarrow \frac{1}{2} = c$ //

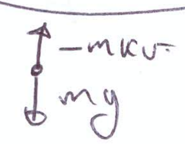
$\therefore \frac{1}{2}y^2 = 4\tan^{-1}x + \frac{1}{2}$

$\Rightarrow y^2 = 8\tan^{-1}x + 1$

$\Rightarrow y = \sqrt{8\tan^{-1}x + 1}$

// Take (+) answer as $x=0 \Rightarrow y=+1$ told. //

(b)(i) Forces



N II $\Rightarrow ma = \Sigma F$

$\Rightarrow ma = mg - mkv$

$\Rightarrow a = g - kv$

…k v, t first

$\frac{dv}{dt} = g - kv$.

$\int \frac{dv}{g-kv} = \int dt$

…r $\Rightarrow -\frac{1}{k}\int \frac{dv}{v - \frac{g}{k}} = \int dt$.

$\Rightarrow -\frac{1}{k}\ln\left|v - \frac{g}{k}\right| = t + c$

$v=0\ t=0 \Rightarrow c = -\frac{1}{k}\ln\left|-\frac{g}{k}\right|$. //

$\Rightarrow -\frac{1}{k}\ln\left(v - \frac{g}{k}\right) = t - \frac{1}{k}\ln\left(-\frac{g}{k}\right)$

$\Rightarrow -\frac{1}{k}\ln\left(v - \frac{g}{k}\right) + \frac{1}{k}\ln\left(-\frac{g}{k}\right) = t$

$\cancel{\Rightarrow \ln\left(v - \frac{g}{k}\right) + \quad =}$

When $v = \frac{g}{2k} \Rightarrow$.

$\Rightarrow -\frac{1}{k}\ln\left(\frac{g}{2k} - \frac{g}{k}\right) + \frac{1}{k}\ln\left(-\frac{g}{k}\right) = t$.

$\Rightarrow -\frac{1}{k}\ln\left|-\frac{g}{2k}\right| + \frac{1}{k}\ln\left|-\frac{g}{k}\right| = t$

$\Rightarrow -\frac{1}{k}\left[\ln\left(\frac{g}{2k}\right) - \ln\left(\frac{g}{k}\right)\right] = t$

$\Rightarrow -\frac{1}{k}\left[\ln\left(\frac{g}{2k}\Big/\frac{g}{k}\right)\right] = t$

$\Rightarrow -\frac{1}{k}\ln\left(\frac{1}{2}\right) = t$

$\Rightarrow \frac{1}{k}\ln\left(\frac{1}{2}\right)^{-1} = t$

$\Rightarrow \boxed{\frac{1}{k}\ln 2 = t}$

(ii) $\Rightarrow$

$-\frac{1}{k}\ln\left(v - \frac{g}{k}\right) + \frac{1}{k}\ln\left(-\frac{g}{k}\right) = t$

$\Rightarrow \ln\left(-\frac{g}{k}\right) - \ln\left(v - \frac{g}{k}\right) = kt$

$\Rightarrow \ln\left(\frac{-\frac{g}{k}}{v - \frac{g}{k}}\right) = kt$

$\Rightarrow \frac{-g}{kv - g} = e^{kt}$

$\Rightarrow \frac{g}{g - kv} = e^{kt}$

$\Rightarrow g = (g - kv)e^{kt}$

$\Rightarrow \frac{g}{e^{kt}} = g - kv$

$\Rightarrow g - \frac{g}{e^{kt}} = kv$

$\Rightarrow \frac{1}{k}\left(g - \frac{g}{e^{kt}}\right) = v$

as $t \to \infty$ $\frac{1}{e^{kt}} \to 0 \Rightarrow$

$v \to \frac{1}{k}(g - 0) = \frac{g}{k}$.